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# Use and exchange value in mathematics education: contemporary CHAT meets Bourdieu's sociology

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**Abstract** Empirical work has shown how mathematics education exhibits certain tensions between its value as being practically useful to production and consumption on the one hand and in offering access to scarce resources on the other hand. These tensions can be ultimately traced to the contradictions in the way mathematical knowledge enhances the use value and exchange value of labour power, respectively. To understand this as a social psychological phenomenon, I look to two well-known theoretical perspectives on education, first that of the Marxist psychology of Vygotsky and activity theory (and contemporary cultural-historical activity theory) which I find tends to marginalise 'exchange value'. Second, I look to Bourdieu's sociology of education that tends to marginalise the use value. I then bring together these two perspectives in a joint theory of education as both development and re-production of labour power, in which use and exchange value both have their place (in commodity production). This helps explain where mathematics education might be critical.

**Keywords** Value · Use value · Exchange value · Cultural-historical activity theory · Bourdieu · Critical mathematics education

## 1 Introduction

Some of our research team's empirical research provoked us to question the values of those engaged in mathematics education and particularly the conflicts between different kinds of values, such as its value in practical use, its purchasing power or 'currency' and the enjoyment of mathematics. Examples of the sort of tensions I have in mind include those between becoming practically competent on the one hand and passing tests on the other, enjoying the subject as an interesting or aesthetic experience versus studying mathematics 'to get ahead in life', learning mathematics cooperatively with others versus competing against and besting one's fellows, mathematics as right-wrong or 'black-and-white' versus

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mathematics being open to discussion and diverse views, learning procedures ‘by heart’ 35  
 versus understanding them via the mathematical concepts on which procedures are based 36  
 and so on. Specifically, in our research into adolescent students’ mathematical experiences, 37  
 identities and practices in transition between compulsory schooling and university we found 38  
 repeatedly that ‘use’ and ‘exchange’ categories seemed helpful concepts (see, for example, 39  
 Williams et al., 2009 and more at *Transmaths.org*). 40

This paper then seeks to understand the values of ‘use’ and ‘exchange’ in mathematics 41  
 education and looks to two central theorisations of education as social practice in the 42  
 literature: First, there is that of the activity theory of Vygotsky, Leontiev and later adoptees 43  
 such as Engestrom; second, there is the sociology of Bourdieu. The former provides a very 44  
 strong perspective on education as a ‘useful’ social activity whose outcomes are 45  
 (normatively) the development of the psyche, especially the development of adolescents 46  
 into adults through their development of adult personalities, including their intellectual and 47  
 emotional capacities (including the disposition and capacity for theoretical and mathematical 48  
 thinking). This view is largely unburdened with a notion of exchange value or of the wider 49  
 sociology of education. Lately, those working in this perspective recognised the contradictions 50  
 inherent within and between educational and other practices due to use-exchange contra- 51  
 dictions. I will argue that these all arise from the central contradiction between the use and 52  
 exchange value of the commodity “labour power” in capitalism and the place of education in its 53  
 (re-)production (see Williams, 2011a). 54

On the other hand, Bourdieu emphasises exchange values or what he calls ‘cultural 55  
 capitals’ in fields, such as ‘educational capital’ in the educational field, and identifies this 56  
 with class re-production: In this view, failure in mathematics is as essential as success, and 57  
 competition and ‘playing the game’ (with the different cards that our class upbringing deals 58  
 us) is universal. But Bourdieu does not have much space for the concept of ‘use’; relations 59  
 of power for him are based on a cultural arbitrary, and the usefulness (e.g. of mathematics) 60  
 is of minor importance to this sort of power. 61

By seeing the value of mathematics as embedded in labour power, I will bring together 62  
 the use and the exchange concepts, and I will suggest that we can draw on both CHAT and 63  
 Bourdieu within a single perspective. The tensions alluded to before are now revealed as 64  
 refractions (in the educational field) of underlying contradictions between the use value and 65  
 exchange value of labour in the economy and hence of class contradictions. The 66  
 explanation for the ebb and flow of educational practice, between progressive and back- 67  
 to-basics tendencies, is therefore rooted in, and an (educational) refraction of, the ebb and 68  
 flow of the class struggle. 69

**2 The problem of value arising in our research** 70

No doubt many of us in the mathematics education community have experience of what we 71  
 believe are progressive, ‘useful’ developments in mathematics teaching and learning being 72  
 sacrificed by political and government authorities, or just falling by the wayside as the 73  
 inertia of the system seems to swallow up innovations and even apparently effective 74  
 teaching and teachers: Assessment and examinations, and wider social and political 75  
 interests, are often at the heart of this retrenchment (Williams, 2010). In such cases, some of 76  
 us have perceived that the development of the education system has in some respects at 77  
 certain times and in certain places even ‘gone backwards’—I have in mind back-to-basics 78  
 movements and policies globally but also the abandonment of investigations, problem 79  
 solving and modelling projects and coursework recently in England, for instance. It seems 80

that critical mathematics education needs to understand better and take account of this wider social, state, governmental and multi-institutional cultural context: We need to understand what we are up against. When I say 'we' here I imply a certain ideological, critical stance on what mathematics education is for, or should be for; I thus align myself with critical trends that argue that mathematics education might help understand and perhaps alleviate ignorance, oppression and so on (for instance, Skovsmose, 1994).

A striking feature of some recent research I was involved in was the way in which values played a key mediating role in learners' and teachers' identity, in shaping choices and pedagogies, and above all in constraining institutional decisions and practices (Hernandez-Martinez et al., 2011a; Williams, 2011b). We came to identify themes related to values: The 'currency' of mathematics grades as a means to gaining access to certain highly valued courses (e.g. medicine) or institutions (e.g. elite universities), or as a means to enhance one's curriculum vita and future career prospects, versus the 'use' of mathematics as a means to understand or practise competently, e.g. in engineering or science. Rather loosely, we called these themes 'exchange' value (relating to the possibility that a grade can be used to 'purchase' entry to a course) and 'use' value (relating to the competence and understanding required to use and apply mathematics in practice), respectively. These terms were not meant yet in the technical, economic sense (to be developed below), yet they connected with the 'exchange value' paid to institution and to the teacher who is performance managed in an institutional culture of payment by results predominating in England in our research study. Then there is the 'enjoyment' of mathematics (which we can relate to the use value of mathematics in the process of consumption). The point is that these various values often came into conflict, causing sometimes palpable distress for learners and teachers (see Williams et al., 2008; Williams, 2011b).

In general, teaching to the test and preparation for high stakes examinations—and what we associated with 'transmissionist' teaching—dominated lessons (Pampaka et al., 2011a). While we found no evidence that this damaged the learners' test scores, we did find an association with declining dispositions towards studying mathematics. The test-taking game of schooling has explicitly and profoundly entered the micro-discourses of lessons with adolescent, pre-university and university 'advanced' mathematics classes of students particularly. Our research suggests that compared to informal assessment through project work, this has a damaging effect on students' engagement with and understanding of mathematics (see Hernandez-Martinez et al., 2011b).

A typical example in my field notes comes from an 'advanced' mathematics class where the students were learning about geometric progressions and series. This class was for the selected students that (a) have survived selective examinations at the age of 16 at a sufficiently high level (i.e. less than 20% of the population take such an advance course) and (b) have chosen mathematics as an option, usually because of its potential value for them in gaining a university entrance qualification (an example of what I call is its 'exchange value'). Thus, the majority of the population have already dropped out by the time the adolescent learner gets to this advanced stage.

The teacher pointed out the formulae for the  $n$ th term and the sum to  $n$  terms of the series with first term  $A$  and common ratio  $r$ , saying: "I could prove this to you but you don't need to know that: they'll never ask you". The class proceeded to use the formulae to find the  $n$ th term and sum to  $n$  terms of the series in routine exercises. For instance, one problem posits a geometric series  $2, x, 18, \dots$  and the problem is to find  $x$ . This was expected to be reduced to an algebraic exercise applying the formulae in  $A$  and  $r$ . Thus,  $A$  and  $r$  are to be found (using equations  $A=2$ ,  $Ar=x$  and  $Ar^2=18$ ) and hence  $x$  deduced. Alternative approaches were not considered or discussed: The concept of a geometric model or mean as

such did not enter the discourse—neither this nor applications outside mathematics are on the syllabus, so neither was considered nor taught. A series of similar but increasingly difficult example problems was presented where each can be solved by reduction to equations using the formulae, and the students tackled these, while the teacher circulated and helped individuals. The teacher used examples that caused trouble to alert the whole class to specific technical difficulties; in some cases, logarithms were needed, for instance. In the following weeks, the students prepared for their end-of-module test, which included such examples that can be solved using this procedure (though the students must discriminate between arithmetic and geometric progressions and retrieve the correct formulae from the sheet provided). By and large if the students have good enough prior grades and the students work hard, the learners pass quite well, and most are happy with their educational progress.

But sometimes as the course modules proceed, there are test questions that are described as ‘harder’ that they say demand ‘deeper understanding’—and these tend to cause trouble. Such questions can sometimes be regarded by students and teacher as ‘unfair’, if they are at all unusual or different. However, in a moment of doubt, the teacher told us that he felt that he was to some extent to blame for such troubles because he—actually he says it is the system as well as his role in it—did not give enough time for the students to develop ‘understanding’. In our research project, we looked at mathematics education across five case study colleges and surveyed approximately 90 teachers and 1,700 students nationally: We concluded that exceptions to this case were very rare. We concluded that the norm was a system focussed on teaching procedures to the test at the expense of understanding, discourse, investigation or modelling and problem solving and that this was encouraged by the whole system of performance evaluation by ‘league table’ results and associated examination and management technologies in England. We analysed this as the influence of the currency or ‘exchange’ value of mathematics qualifications for the various stakeholders and institutions: students who need the grades for university entrance, schools whose reputation (and funding) depend on their students’ successes and teachers whose performance is carefully line-managed in the light of assessment data (for details of this classroom and analyses of the learning and pedagogy, see Williams, 2008; Pampaka et al., 2011a; Pampaka, Williams, & Hutcheson, 2011b).

But now, in our most recent Transmaths project, my colleagues and I followed students across the transition to university courses: Here one finds academics complaining that students have been taught instrumentally—‘understanding’ is now at a premium, and even students with good grades may find the mathematics expected of them very demanding, perhaps too demanding. While their grades got them into a course to study a mathematically demanding subject, the use of their mathematics often then proves suspect. Many of the mathematically demanding courses we studied (across 13° programmes in five universities in England) found ways to help students who struggled with the mathematics components of these courses, at least in the first year, providing extra support and even one-to-one tuition if necessary and thereafter helped them to avoid more challenging mathematics components, providing a degree pathway, grade or title that allowed flexibility as to their quality of mathematics. At its most extreme, however, the university system may fail the student at this point, but in practice we found such threatened students looking for other ways forward, finding an alternative university or degree course (see Pampaka et al., 2011b for some analysis of the transition).

In one case, we found a student who had chosen a ‘hard’ science course mainly because they thought it would look good for their vita/CV, and when they found they could not cope, they opted for an alternative career path in the army. Of course such students lead to

**Q3/Q4**

loss of income for the institution, and such losses are a concern for university management. 179  
 Too high a dropout rate also may draw attention and loss of value in other ways too; it has 180  
 to be said that our research found it remarkably difficult to track down data on such drop 181  
 out because the institutions do not make such data easily available. 182

Thus, at university too the conflicts between the currency and use of mathematics 183  
 continued to mediate decision making by learners, lecturers and managers. The question is: 184  
 how to understand these values and value conflicts from a critical perspective? One could 185  
 consider a number of theoretical foundations from which to approach this; the concepts of 186  
 exchange and use led naturally to Marx and to two perspectives on education that draw on 187  
 these concepts. In this paper, I will examine this problem from the theoretical perspectives 188  
 of Marxist social psychology (Vygotsky and Leontiev's CHAT) and of Bourdieu, 189  
 respectively, with a view to illuminating this problem itself, but also leading to a 190  
 comparison and contrast between the two theoretical perspectives. 191

**3 Cultural–historical activity theory** 192

The corpus of cultural–historical activity theory we refer to begins with the attempts of the 193  
 Russian troika (Vygotsky, Luria and Leontiev) to create a Marxist 'cultural psychology', 194  
 apparently supported philosophically by a re-examination of Hegelian and Marxian 195  
 dialectics, inter alia by Lenin and Ilyenkov (see Ilyenkov, 2009). Since this work was so 196  
 often partially 'read' in translation in the West, it will be necessary to revisit it, especially 197  
 insofar as concepts of 'value', education and labour are concerned. I will argue that a 198  
 misreading of Vygotsky fundamentally turns his psychology from 'in the beginning was the 199  
 deed' to in the beginning was the word meaning (dangerously close to conscious or verbal 200  
 'thought' in the idealist tradition Marx criticised!). For Vygotsky, as for Marx, the 201  
 motivated action/deed in practice is primary, and consciousness then arises as the mediator 202  
 leading to new actions, thus: action–thought–action (Vygotsky, 1986). Therefore, as Lenin 203  
 and Ilyenkov develop the Hegelian thesis, logic/Logic is the necessary outcome of human 204  
 engagement in practice: Even the law of distributivity of 'and' over 'or' is held to be a 205  
 necessary consequence of practical engagements of millions of humans in activity engaging 206  
 with the objective—natural as well as social—world. The essence of the cultural–historical 207  
 perspective and the genetic law as Vygotsky has it then is that social practice is the source 208  
 of consciousness, especially all higher (volitional and verbally mediated, logical and 209  
 conceptual) mental functions, and so in the development of mind it is cultural activity that 210  
 pre-figures the development of the higher cognitive functions, including all language and 211  
 mathematics. 212

Vygotsky (1978, p 52) quotes Lenin on Hegel (1961, pp 190 and 217) as follows: 213

It (Hegel's syllogism: JW's comment) has to be inverted: the practical activity of man 214  
 had to lead his consciousness to the repetition of the various logical figures thousands 215  
 of millions of times in order that these figures could obtain the significance of 216  
 axioms... Precisely (and only) on account of this thousand million fold repetition, 217  
 these figures have the stability of a prejudice, an axiomatic character. 218  
 219  
 220

But we note for Vygotsky and Luria, 'everyday practice' was the birthplace of everyday 221  
 conceptions and of common sense logical thinking and so in many cases inadequate and 222  
 certainly inadequate for scientific conceptions and practice. As Marx argues, if everyday 223  
 surface perceptions of the world were sufficient, then we would not need science at all 224  
 (Vygotsky, 1978, p 173 quotes Marx on this). For Vygotsky and Leontiev, then schooling 225

and schools exist to engage learners in humanity’s culturally most advanced, scientific knowledge, conceptions and thinking, through scientific practice. Ultimately (in the adolescent stage) this implies engagement in scientific practices from which ‘theoretical’, ‘true’ or ‘scientific’ conceptions might develop. Science/theory is here understood to be the hallmark of modern human culture and hence modern consciousness held to be prerequisite for socialism. Science and scientific concepts allow one to go beneath the surface, beyond superficial or purely empirical understandings of everyday practice, to build and work with theoretical models that yield robust, science-based implications in and for practice. For me, this gets to the heart of the ‘use value’—or potential use value—of mathematics education as a tool for ushering in scientific, theoretical thinking and practice, i.e. scientific labour.

But here please note: One can use the term ‘use’ loosely, as we did above to describe our empirical work, but for Marx ‘use’ was also an attribute of commodities under capitalism—their use (in the ordinary sense now) being realised in consumption. I have argued elsewhere and here build on the argument (Williams, 2011a, b) that mathematics is not only useful in this everyday sense but actually technically in Marx’s sense becomes a ‘use value’ of mathematically enhanced ‘labour power’, a rather special commodity owned by the worker, rather than the capitalist, but partially ‘consumed’ in capitalist production (i.e. it is consumed as labour time but uniquely produces more than the value required to reproduce it: hence surplus value and profit). As such mathematics also acquires the exchange value, attribute of this commodity and use and exchange value come into classic Marxian theoretical contradictions. Class interests, for instance, emerge in the production of labour power... but more of this later.

How did Leont’ev (1978, 1981) conceptualise activity/consciousness and the development of the mind in adolescence? First of all, one must understand ‘activity’ as both concrete material acts and actions in a social practice (its observable, material and objective form) and as a movement of the human psyche, as ‘goal’ and ‘motivation’ (it is ideal, human subjective and intersubjective form). Leontiev is explicit that an activity is essentially determined by its ‘object’: Without an object, there is no activity, and the object consists of the dialectical opposition of the material stuff to be transformed by action/activity together with its idealised, humanly envisaged outcome that is its goals and motivation. Labour engages a collective in joint activity (with a collective ‘motive’) realised in practice through a series of individual (‘goal’ directed, conscious) actions enacted in order to transform and change the object, but only in order to effect a transformation of the object into a previously envisaged outcome: This is the true ‘motive’ of the collective activity. Thus, when a building company makes a house, there is a series of actions on materials, each one goal-directed and an essential part of the collective, whole activity whose motive was previously envisaged as a house, based on some ideal model or plan of the house to be built. Note the importance of motives, goals, imagined outcomes and thus emotions and needs to be satisfied. But also notice the material form of actions on objects. For mathematical practices to become meaningful from this perspective, activity requires an envisaged outcome, plan or design that meets some real need, in practice. In the ideal case (see my account of Leontiev’s ‘primal hunt’ in Williams & Wake, 2007), the various conscious goals of all the individual actors/actions are well coordinated with the social, collective ‘motive’—but in general in modern life, there are contradictions between different conscious goals and the social motive, due to a complex division of labour; these contradictions provide for the process of change and development, both of the activity as a whole but also for the consciousness and goals of individuals involved.

Vygotsky and Leontiev probably developed these notions most fully in the context of their joint work on pre-school play, but here I want to adopt their presentation of adolescent

Q6

Q7



activity including 'schooling', which often appears quintessentially semiotic. Leont'ev (1981) describes the issue of motives and the object of activity in schooling as follows: A student is reading, nay studying, 'acting on' an object, say a school history text (the reader may translate: for history read mathematics): We can see the operational act of reading, and we can infer the conscious 'goal' of the student's action is to learn some history, but what is the social content of the activity involved, what is its social motive? (For a more detailed account and case study, see Black et al., 2010).

Let us see what happens when the student's friend asks him why he is studying this text— does he not know that this text is no longer on the exam syllabus? Leontiev considers two possibilities here. The student may cast the text aside in disgust (the motive of their activity was apparently 'schooling', based on exam performance and winning prizes). Alternatively, the student is reluctant to leave the text, or continues their study: In such a case, the act of studying history may have acquired a new motive and so be constituted as part of a new, different activity. Perhaps the student will say they are 'studying history for its own sake'. Leontiev hints that the meaning of the study of human history has some deeper social motive or human essence, perhaps for an understanding of the human condition, its prospects for survival etc. Indeed, one now recognises how the adolescent can develop new motives and new activities from schooling: One may begin with one motive/goal but even without being consciously aware shift to another, more grown up, more socially meaningful and essential motive. Leontiev argues then, after Vygotsky, that 'development' involves such personal growth: Indeed, the definition of person-hood or self (they actually used the term 'personality' which I will avoid here for obvious reasons) they gave was the hierarchy of activities that a person engages with.<sup>1</sup> Thus, the development of the mathematical psyche of the adolescent, in this view, involves engagement in the most advanced, most valuable, collective, mathematical activities possible, and motivation is the single most critical issue. As Vygotsky has it:

The tasks with which society confronts an adolescent as he enters the cultural, professional, and civic world of adults undoubtedly become an important factor in the emergence of conceptual thinking. If the milieu presents no such tasks to the adolescent... his thinking fails to reach the highest stages, or reaches them with great delay. (Vygotsky, 1986, p 108).

Now we can make sense of Vygotsky's dictum that all 'good learning is in advance of development': The purpose of schooling for the adolescent is to lead their development through engagement in new, more culturally advanced, collective activity, engaging with new more developed, social motives that transform school actions into more socially and culturally meaningful activity.

Recall that for Vygotsky, the adolescent phase of development was characterised by (a) a shift in the whole arrangement of their intellectual functions around new motives (adult sexual and other relationships, work etc.) and (b) the principal new interest that school could offer being logical thought and theoretical, or scientific, concepts which raise the status of the adolescent's thinking to that of an equal to the culturally most developed adult. It is vital to recognise two features of this: First, Vygotsky was acutely aware that school concepts tend to pure 'verbalism'. For him, after Hegel and Marx, a 'true' concept is only

<sup>1</sup> I am aware that there is scholarship that points to some relevant differences between Leontiev and Vygotsky and even between Marx and Engels: For the purposes of this paper, I avoid such controversies, and this account sticks to material on which they appear to agree.

complete when it ‘ascends to the concrete’: that is, a conception must be made meaningful by virtue of its work in practice, i.e. in activity. Thus, the entire schooling, work-to-the-test, would be pure verbalism and not ‘true’ conceptual thinking. Second, even in the revolutionary times, Vygotsky lived in—let alone those of Leontiev—school activity tended to become separated from the ‘real’ more culturally advanced activity that adolescents might be motivated to engage with: There was therefore always the danger that motives would not be engaged and students’ development would be frustrated and become ‘dysfunctional’ (Leont’ev, 1981). In fact, as Vygotsky recognised, the full engagement, or better, synthesis of education with labour would have to await socialist development: Perhaps the ‘new man/woman’ of communism might overcome these splits and contradictions.<sup>2</sup>

New generations and new forms of their education represent the main route which history will follow whilst creating the new type of man... Collectivism, the unification of intellectual and physical labour, a change in the relationship between the sexes, the abolition of the gap between physical and intellectual development, these are the key aspects of the alteration of man that is the subject of our discussion. (Vygotsky, in van der Veer and Valsiner, 1994, pp 181–182).

To sum up then, Vygotsky and Leontiev would interpret the emphasis we have noted on the currency or exchange value of mathematics as an exercise in getting the grades on the test as a failure of the institution to develop the motives of mathematics beyond schooling, as ‘pure verbalism’, and as dysfunctional. They would, I infer, have seen the work of overcoming this dysfunction as an essential task of their revolution.

Davydov and his followers in subsequent generations added a key notion for us: The idea that mathematics adds some key features to that of theoretical concepts and thinking—especially through mathematical models and modelling. Davydov (1990) notably re-conceptualised Krutetski’s notion of the ‘gifted mathematician’: He argued that ‘all’ that was involved in the so-called gift was genuinely ‘theoretical’ thinking in mathematical problem solving and that with appropriate learning all students could and should be brought to this level of development.

Subsequently as translations of Russian CHAT literature began to reach the West, Cole, Engestrom, Werstch and others began an integration of this work—along with the Bakhtinian school—into educational and social research, where ideas of a new generation of CHAT emerged. Engestrom (1987) is widely cited as one principal synthesiser in this later era. Inter alia, he gets to the core of the dysfunctionality of schooling and identifies its primary contradiction—between the use and exchange value of the commodified objects of school education (mirrored in all public services where accountability is mediated by measures that give a cash equivalent form to the learning outcomes: see Power, cited in Williams, 2010). Thus, the value of objects of schooling—mastery of texts etc.—when divorced from their use in practical activity becomes reduced to a currency symbolised in grades, exam scores and all the distortions that accompany such a system. Engestrom specifically refers to use and exchange value in the schooling and health contexts. But we must again be careful of calling this ‘exchange value’ in Marx’s technical economic sense: The certificates and grades students are awarded do not have ‘use’ to be consumed in themselves and cannot be actually ‘exchanged’ as such; rather, they are symbolic of some

<sup>2</sup> Having said this, educational psychologists in the Soviet Union that raised such social/sociological dimensions and issues were prone to losing their chairs or their heads: This whole side of theorisation was therefore underdeveloped in AT in the USSR.

value that inheres in the individual awarded the grade or certificate, which ultimately refers to the value of their labour power, which is a commodity properly speaking [Thus according to Marx, the source of all value is labour power and labour time, a commodity bought from the labourer with wages by the capitalist entrepreneur who uses it to create commodities which embody a surplus value and thus sell at a profit.]

The question of overcoming the split between education/schooling and practice/labour was explicitly taken up by Engestrom (1991): This remains a key feature of activity theory and CHAT in education today. Particularly in mathematics education, an important focus for those inspired by Vygotsky or CHAT has been the relationships between mathematics in school and activity in the home, in everyday, or in workplace mathematics. Additionally, key themes have been mathematics made concrete, or crystallised in artefacts and tools, and mathematics made powerful in theoretical–scientific–technological practices. Typically, critical (mathematics) educators argue for a kind of (mathematical) literacy: for (mathematics) education to help youngsters perceive how knowledge (mathematics) can be powerful and can help them to understand and control their social and emerging adult lives (e.g. Skovsmose, 1994).

More rarely, critical mathematics educators have pointed to how mathematics education is often, perhaps mostly and usually, in practice, alien and alienating for young students and adolescents. Lave and McDermott (2002) pointed out—drawing on the Marx's early essay 'Estranged labour' in his 1844 manuscripts—that the educational establishment has made learning alienating just as the factory made labour alienating. This is more than an analogy. As I argued (Williams, 2011a, b), the state institutionalises 'learning' in schools as a system for manufacturing new generations of labour power, and this necessarily implies alienation of the learner to the extent that they learn to labour, that is, they learn to enhance their market status and labour power for a labour market. This then prompts an analysis of the products of education as commodities, albeit special kinds of commodities like labour power. The students makes themselves into commodities by treating their labour power as a commodity for the labour market, their credentials as symbols of the enhanced, 'distinguished' exchange value of their labour power and their learning as an investment of their time and energy in acquiring value, particularly in competition with others in preparation for a competitive labour market. This competition with others ultimately alienates the learner from other learners and even from their own learning and so from themselves.

Of course there remains the contradictory 'use' element of labour power here too: Labour still has use value to the extent that it will be consumed in the labour process in making commodities and so must be useful and usable before it is in any way marketable. Ultimately in the classic Marxist analysis of the competitive labour market, mathematics must prove its potential use if it is to continue to command market value. Additionally, however, if mathematics enhances labour power, it also, in Marx's analysis, may lead to the undermining of the system, as it may enhance the power—particularly in this case the intellectual power—of the proletariat. This is the central contradiction that can give critical educators some hope: If mathematics enhances intellectual labour power, it may also enhance the capacity of labourers to be critical.

However, this is not 'all': The liberal view of education as the development of citizenry more broadly should not be denied its place here. The next generation of labour learns also how to behave, how to consume and how to 'do' leisure (all riven with the same primary contradictions between use and exchange). For Marx, labour and production are at the forefront, but re-production, consumption, distribution and exchange are all integral parts of the economic process. As Althusser and others argue, capitalism would not have survived

and would not continue to survive, if it did not continually reproduce the conditions required, and this includes the re-production of a requisite cohort of labourers, on a generational as well as daily timescale. We need to understand this process in more depth than the previous discussion offers: How does the economic (production) system shape the re-production (education) system?

In the context of education and other fields of re-production, Bourdieu is the leading protagonist in theorising cultural aspects of what he calls ‘capital’. If we are to understand how the education system gives cultural value to mathematics, we have to understand its cultural ‘capital’.

**4 Bourdieu’s sociology of education**

In the above narrative, it took us a long time to get to mathematics education as a field of production of some kind of ‘exchange value’ because labour power is not a commodity like other commodities, sold by capitalists on the market. In fact it is not usually sold at all until education is largely over—and the state is clearly central in mediating this re-production (as Marx says, in part protecting the next generation of labour from the rapaciousness of the individual capitalist’s tendency to exhaust its workforce). For the most part in the CHAT perspective, the ‘use’ of education as a process of the cultural development of the child was prominent even if implicit. Now, with Bourdieu, use value will appear to disappear.

Bourdieu’s sociology goes straight to the process of exchange and of the value of ‘cultural’ capitals in the fields he analyses (including education) and of their exchange rates in the field of power, ultimately economic and monetary power in capitalism. For instance, economic capital helps the dominant classes to develop their children’s educational capital: They spend money to help advance their educational careers, the children and parents thereby work to exchange economic capital into ‘educational capital’. Already at school entry, we know that middle class children are better attuned to school activity and discourses by virtue of their parents’ pedagogic work: In part, this is a function of the family’s economic as well as cultural capital, not only having the spare time but also the educational competence to prepare their children. Bourdieu says that their children have the appropriately structured habitus (especially by virtue of their linguistic capital, but in later years mathematics capital becomes prominent) that gives them a natural ‘feel for the game’ of school, i.e. that their habitus matches the structures of the educational field.

For him, the educational field is a system of structured, structuring relations in which power is determined by the relative cultural ‘capital’ accorded to positions—or individuals who occupy these positions—in the field. Such capital is constitutive of the ‘arbitrary’ superiority of the dominant classes and is reproduced from generation to generation (Bourdieu & Passeron, 1977; Bourdieu, 1977, 1998). To the extent that the educational field of power is arbitrary, i.e. to the extent that it excludes the dominated from access to educational capital and, eventually, desirable social positions, it inflicts a ‘symbolic violence’ on them. His close analysis of the French education system (in the 1960 and 1970s) shows how the lower classes were indeed systematically excluded. His statistical analysis reveals the ‘objective’ facts of class exclusion (with significant individual exceptions ‘proving the rule’) and the importance of linguistic capital in the process.

But his ethnographies also reveal the subjective side of the habitus that comes to be preferred or marginalised: In the main, the dominated habitus comes to feel that the classroom or university is ‘not for the likes of us’, but—because there are notable exceptions—this is accepted as by and large their own choice or preference—the violence is

inflicted with their tacit consent and so is 'symbolic'. Willis (1981/1977) elegantly showed how the last thing the 'lads' want to become is like the passive, effeminate, middle class kids who listen to every word of the teacher—who they called the ear 'oles (nowadays in England they might be called 'swats' or maybe even geeks).

A particular aspect of Bourdieu's analysis of the educational field was the contemporary significance of linguistic capital: Thus, the competence to write a certain kind of essay, with 'flair' and 'originality', was what got you ahead in the educational 'game' at the top level. These things cannot, so goes the dominant view, be simply 'taught'. Thus, the very competences that schools deny to the students are in the end 'what counts' in the top examinations and what is distinguished at the highest level. But we note also that Bourdieu in his later work (e.g. 1987/1990, 2000/2005) suggests that it is mathematics (more than language as such) that has become the marketable, educational 'capital' that many students need to get access to the preferred courses and higher institutions. Thus, mathematics becomes a new cultural capital of this educational field, which inflicts symbolic violence to the extent that it reflects a power that is arbitrary, and hidden, or 'misrecognised'. It is not the only such form of capital, however: Among the elite of elites eschewing the sciences can be a signal of distinction (one notes that fewer scientists than ever were recently elected to parliament and that the top tier of government rather reflects philosophy, politics and economics at Oxbridge).

Another key insight in Bourdieu's reflections on education is his emphasis on the examinations that select: It is absolutely important ideologically that the school system is perceived to be 'fair' and equitable at the same time as it inflicts symbolic violence on the dominated (Bourdieu & Passeron, 1977/1990). In fact, this is the very point of 'symbolic' violence per se, i.e. that it is not (normally) recognised as such. The doxic experience of education—doxic insofar as it works because it is implicit and must not be exposed—might be explicitly formulated as follows, i.e. as 'orthodoxy': Those that have acquired mathematical competence can benefit themselves and others, and society in general, by being selected to learn more and contribute to the higher echelons of society. By the same token, those that have not acquired such competence, rather than being helped to acquire it, shall be deemed unfit as such and encouraged to see themselves better off elsewhere, or justifiably cast aside in extremis (maybe to the vocational courses or humanities ..., or worse, God forbid, education).

But the most important structural feature of this 'violence' is that it cannot, or must not, be recognised as arbitrary: The examinations must be seen to be fair and equitable, and this is one of the most important bastions of the system that must be protected at all costs, against all other competitive calls. All to the better if the students themselves come to recognise (that is 'misrecognise'—in Bourdieu's terminology) that they do not have the inclination for mathematics, that they will be happier elsewhere than in the rigorous and demanding—the most elite and selective—institutions, what with their ascetic ways, intellectual 'cold baths' and their associated strange, often seemingly disgusting, styles of consumption.

In all this, though, we do not see a concept of 'psychological development' through education. There is little psychology in this account in general at all, and there is no 'use value' in learning and teaching in adolescence, as in CHAT. This is all about exchange, the market, competition and domination. In short, despite Bourdieu's objections to being called a Marxist and his critiques of Marxism, his theoretical position is that of the critic of the bourgeois education system as essentially a reproducer of the class system, albeit in ever more sophisticated cultural forms that serve the, always essential, misrecognition. Having searched through many Bourdieu texts, I have yet to find more than a few almost trivial

references to the 'use' of education or knowledge for society and production per se, though there are occasional references to the 'no doubt' technical value of education to 'competence' and the efficiency of production. His point is that qualifications that distinguish and privilege the elite are essentially arbitrary in that they maintain the arbitrary domination of the elites but in some fields must carry the functionality that serves to maintain the doxic beliefs that the elites are not in fact arbitrary. Thus, functionality is always juxtaposed with cultural capital (so refracting economic capital and exchange value) and its associated symbolic violence. Thus, referring to the 'nomination' of the elites via titles and accreditation in the education system:

In fact, the technocratic illusion is partially justified, and .. the effect of misrecognition at the basis of the magical efficacy of titles, and .. the symbolic violence of all acts of nomination, is only possible because titles also certify the acquisition of technical skills... Titled individuals are legitimate titulars of exceptional positions, but to a certain extent they also possess uncommon technical competences, which provide a foundation for their monopoly. We also note that the market value of a title, however fully it may depend on the power of symbolic imposition, is always partially determined by the scarcity of the concomitant technical skill in the market.

Having said this, we cannot establish once and for all and for all cases how much of each of the forms of academically guaranteed competence is strictly technical *skill* and how much strictly social *dignity*. (Bourdieu, 1989/1997, pp 118–119)

But then, according to Bourdieu, the rules of the game that must not be spoken can come to be spoken about. How? Through his own sociological analysis! Alongside all this 'contemplation' of society, then there is an activist—albeit a reflexive–sociologist–activist—at heart. Bourdieu sees himself as one who works to bring the unsaid and unsayable into the range of discourse, to make the doxa challengeable and to take conscious control of the most powerful aspects of the habitus (its hidden, unconscious side). In fact, he stresses the role of the scientist of the practice, 'outside' of and objectifying the practice: Only through such science might the objective facts about the true relations in society be made knowable (Bourdieu, 1996). Only by escaping the worldview of the practitioner caught up in the flow of the game, only by standing aside from the habitus of those that know and feel the game from the inside, can one find the objective essence of the field. In this, one might argue that he was well aligned with Marx's concept not only of the radical intellectual but also of the Vygotskyan and CHAT (as in Davydov) perspective on what constitutes 'theoretical', in contrast to 'everyday', concepts, as described above.

What is more, Bourdieu finds a powerful role for mathematical analysis in his own statistical work—in his own methodological practice as a critical and reflexive sociologist. He thereby makes clear for us that mathematics is crucially useful for escaping the misrecognition necessarily involved in everyday subjectivity, for instance by helping to uncover and objectify the gruesome, statistical facts of exclusion of the dominated in the educational process. I cannot find this reflection anywhere in Bourdieu's writings: Almost nowhere can one discern that education after all provides him (and presumably then others) with the useful capacity of mathematics that may allow one to penetrate into the essence or truth of society and so to a science of society. Nor does Bourdieu deal much with consciousness, a crucial aspect or 'moment' in activity according to CHAT and surely crucial to understanding how economic and cultural values become internalised and hence mediate learning and motivation.

To sum up, Bourdieu offers us a way to understand how the field of power and the economy come to structure social fields such as education and how the economy and the dominator-dominated relations are refracted in power relations and positioning in the educational field. This complements the theory of CHAT that provides a conceptualisation of learning and development of the adolescent into a labourer and the possibly critical contradictions in values that are thereby internalised. The task is to bring these two complementary perspectives to bear on the theory of educational values.

## 5 Comparison, contrast and synthesis of CHAT and Bourdieu

I am not the first to look to bring CHAT and Bourdieu into a combination or a synthesis. In search of a theory of identity, agency and culture, Holland, Lachicotte, Skinner and Cain (1998) draw on aspects of Vygotsky, Bakhtin and Bourdieu (and in fact modern work on cultural models more widely) in their theory of Figured Worlds. Their concern is with the development of a notion of 'identities in practice', and they begin with an enunciation of Vygotskian activity theory proper in regard to practice, activity, semiotic mediation and the 'zone of proximal development'.

In their view, Bourdieu's concept of the field needs Vygotsky to account for how the societal enters the subjective psyche, and this is what Vygotsky's semiotic mediation and internalisation offers. Because the positions in the (presumably 'external', though ideal as well as material, objective) field come to be 'symbolised' in Figured Worlds, they are in practice internalised, and in Holland et al.'s terms, they become culturally 'figured'. The concept of figuration here is, in our view, intimately connected with language, imagination and the mediating, ideal objects of interaction: Thus, they link between the external and the internal—they structure and mediate both the outside/intermental and inside/intramental—in one concept, that of Figure (very much as Vygotsky did with the unit of speech and thought termed 'word meaning').

However, their particular synthesis does not meet the task in hand here, namely understanding value. Recall: We seek to explain the way that the use and exchange values of mathematical labour power in society is produced and come to mediate the activity of mathematics education. CHAT already (credibly) claims to be a social psychology, but I argue that, compared with the analytical framework of Bourdieu's capitals, its sociology is underdeveloped for our purposes. That is, there are sociological assumptions implicit in CHAT—or much of early CHAT at any rate—that need to be elaborated; the foundational assumptions neglect the ongoing success of capitalism and the significance of the educational field as a reproducer of social relations rather than a tool of social liberation. Bourdieu's theory provides a sociology that addresses some of these key concerns: It is especially insightful in terms of how the currency (refracted exchange value) of mathematics functions as educational 'capital' and how this is shaped sociologically in the educational field(s) and its contemporary institutions. But having said this, Bourdieu seems to marginalise the potential of mathematics to help develop scientific, theoretical thinking that might help counter orthodoxy and even support critical reflexivity. I argue that critical educators need to make explicit the use value of mathematics to critical sociology and ultimately to the formation of class consciousness by enhancing the intellectual power and critical faculties of adolescents and future labourers.

Our motivation in the introduction leads us now to a synthesis of Vygotsky and CHAT's tradition with Bourdieusian sociology in a combined theory of educational value. Starting with CHAT and working into Bourdieu, then one might look to an extension of CHAT to incorporate Bourdieu's sociology in the style of Holland et al. This is a sociological extension of the model of

education that can incorporate the CHAT perspective on the ‘cultural development of the mind’, which rests on the ‘use’ of mathematics as a tool for the critical, scientific examination of society (as well as nature). Such a move must, however, recognise the importance of the fundamental contradictions within mathematical labour power as a commodity: As such, mathematics provides a cultural arbitrary for exchange just as much as it provides use value, it constrains as well as affords and it inflicts symbolic violence and orthodoxy (think ‘derivatives’) as much as it reveals, or serves to construct, objective social truths.

On the other hand, starting with Bourdieu and working into CHAT, as it were, the development of mind through education might become the social–psychological expression of the Bourdieusian critique of re-production of the dominant social relations. Thus, the purpose in developing the mind of the student is, *inter alia*, to develop the critical reflexive sociologist. In this case, the practical work to be done is to engage the dominated with the mathematics of reflexive sociology, by making the field of power in its many manifestations an object of mathematical and theoretical study.

In either case, what emerges for us is the dual but contradictory value of mathematics education in the re-production of labour power—as an exchange value of capital in the educational field *and* as a use value in the development of mind, i.e. of the consciousness of the worker. What holds back the progressive potential of mathematics as a theoretical tool is the very arbitrary exchange value it commands and in virtue of which it mediates symbolic violence on the dominated, especially through their apparently voluntary self-exclusion from mathematical activity. Critical educators can challenge this: by encouraging accessibility of mathematics for all (actually ‘de-valuing’ mathematics by making it ubiquitous), by arguing against competition and selection, by campaigning for more ‘time’ for education, by arguing for and developing ‘use’ of mathematics and by directing mathematics at contemporary social problems (climate change, economic and social collapse, exploitation, slave trade, corruption, racism, sexism etc.).

But the foregoing analysis suggests that critical mathematics educators will find resistance on all these fronts and from many quarters. The contradictions in the values involved will lead forces of domination to emphasise competition for scarce resources, ‘freedoms’ of the privileged, separation of education of the elite from the mass and so on. Behold: class warfare in activity in educational field and in the consciousnesses of the students and workers engaged there.

**6 Conclusion**

No doubt there will be those within the educational research CHAT community who will see Bourdieu as an unnecessary accoutrement and even more probably vice versa. But this argument has suggested that there are weaknesses in each that beg a synthesis of both perspectives. My argument here is that this is done by understanding the CHAT perspective on social psychology within a Marxist and Bourdieusian analysis of education as re-production of capitalism and the perspective that the educational field thereby refracts the economic (use and exchange) value contradictions into class contradictions mediated by cultural, educational ‘capital’ in Bourdieu’s sense. Educational ‘capital’, I argue, has its ‘use’ and ‘exchange’ value that can become economic ‘use and exchange value’ when it enhances the commodity labour power. The enhancement of this educational ‘use’ value is the subversive element that threatens capitalism: It belongs to the labourer and can potentially be applied critically to expose the truth of domination in the manner that makes Bourdieu’s own work critical.



In practice then, the CHAT activist may be faced with the question of the geometric series as an enhancement of the use value of labour and the labourer (and of consumption and the consumer). The critical teacher, teacher educator, text author etc. of the geometric series might face the need to model phenomena such as interest rates, financial bubbles such as property and oil, the banking crisis, the power of derivatives and the cost of private financing of state services or the limits of natural resources and consequent climate change. But then, ultimately an analysis of the field of power in education, health and government are at issue in all these, Bourdieu's sociology implies.

For both these perspectives, mathematics must be understood as a theoretical tool, as a model for real tasks and problems in practice. Its concept meaning becomes absolutely vital, of course, and the formulae and procedures we teach are a minor part of this, if not a distraction. A critical approach to the problem of the geometric mean of 2 and 18 could take many approaches and mathematical forms (all symbolically equivalent to the equation  $2:x=x:18$ ), but all of them have at their heart the theoretical concept of common ratio and thus the discrete form of the multiplicative exponential function involved in growth, bubbles, interest etc. and their limits. The 'use' of the geometric progression is in mapping across all these contexts and seeing them as, in a sense, i.e. in a mathematical sense, the same.

Activism then requires a challenge to the instrumental forms of mathematics predominating in the contemporary state education sector we studied. But on the other hand, we must also recognise *pace* Bourdieu that the field will deflect this activism and that the exchange value of mathematics in the educational field will focus the aspirant learner elsewhere, in the preparation for examinations and the acquisition of grades in competition with, rather than cooperation with their peers. Currently, this serves most individual learners ill, as they only subsequently find out they may have been playing the wrong game. But of course ultimately this is not the point, as the point is that the arbitrary nature of the educational competition serves to deflect the learners from coming to critically understand the world they face collectively.

It may even be that the educational field—at least as it is institutionalised in schooling—is not the best field in which to practise education as we critical educators would like to know it. But this conclusion would reflect a defeat of critical education and may not yet be everywhere justified.

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