Mathematics on the Inside: A Review of *Loving + Hating Mathematics: Challenging the Myths of Mathematical Life*

Loving + Hating Mathematics: Challenging the Myths of Mathematical Life (2011). Reuben Hersh & Vera John-Steiner. Princeton University Press. 416 pp. ISBN 978-0-691-14247-0 $29.95

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Imagine you have woken from a dream, a dream brimming with meaning, with passion, with mystery. You try to sustain the feeling, recount the details, share the experience. You fail. Your powers of reconstruction too meager, your tongue too clumsy. Mathematics is such a dream, dreamed by individuals, personal, yet remarkably in a waking state, and provoking sufficient commonality in its recounting to bring individuals together, to create a community of shared passion. The first dreamer of the dream, shrouded in history and myth, perhaps was Thales of Miletus, who later advised Pythagorus of Samos who subsequently founded an order of adherents holding knowledge and property in common while pursuing philosophical and mathematical studies as a moral basis for the conduct of life (http://en.wikipedia.org/wiki/Greek_mathematics; http://www.mathopenref.com/thales.html).

Now, thousands of years later, imagine that you, like Reuben Hersh, have lived your life within the great ensuing culture of mathematics, animated by that shared dream, enmeshed in the politics, the triumphs, the animosities and friendships of your time. You look back on your own life spent contributing to the body of mathematics proper, and voicing its unique and fabulous culture for a broader society that supports it in near total ignorance of its inner treasures. The experience of mathematics and of a life lived in mathematics begin to meld. The community of mathematics itself becomes precious, its trials and tribulations, peaks and valleys of your own experience, its characters animating your subconscious. As a dream, you begin to sense your own era slipping away, inevitably, into the shrouds of history. Is there a moment of panic? Perhaps a moment. But if you are Reuben Hersh you solve problems, not succumb to them. You rally your memory, not only personal memory but the collective memory of your time as recounted in memoires and biographies of your contemporaries. With the assistance of your wife–who better to help than Vera John-Steiner, herself a renowned scholar of creativity and sociohistorical theory–you map out a structure to house the memories, the heartbreaks, the triumphs; a structure to help shape the way in which the next generation of mathematics understands its received legacy. Then you lovingly fill it in with the stories, the stories, the stories, the fabulous stories of mathematics in your time. That collection of stories is *Loving + Hating Mathematics*.

The audience for this volume is anyone who seeks to understand and appreciate mathematical culture, especially those who love mathematics and intend to support its furtherance. For mathematicians it is a guideway to the world they inhabit, or are entering. For educators, it is a summation of mathematical values toward which schooling might aspire. For psychologists and sociologists it is an honest and informed cultural self-appraisal by a savvy insider.

It is not my purpose to lay out the thematic content as framed by the authors; the introductory chapter available from the publisher at http://press.princeton.edu/chapters/i9283.pdf does a fine job of discussing four prevalent myths about mathematics that organize the book, asserting what instead is the true state of affairs: mathematics is a passionate affair (chapters 2-4); mathematics is a social undertaking (chapters 5-6); mathematics has a place in it for contributions of all races, all genders, and all age groups (chapters 7); and mathematics achieves its true educational value when not made mandatory–needlessly–for passage into other professions (chapters 8-9). Instead, I take the opportunity of this review to pull out one of the crosscutting themes that animated this book for me, and that reveals its depth and importance.

Mathematics: A Complicated Place to Call Home

This book is about the passion of mathematics, a passion bounded on one side by hyper-competitiveness that ultimately undermines the ideal of a shared culture of intellectual collaboration and on the other by the dissociation of madness. In between is the vast productive space of creative contribution in which "mathematicians rely deeply upon each other to evaluate their proofs and findings even as they compete for prizes and fame" (p. 84).

Competitiveness has a long history in mathematics in the form of challenge problems that mathematical masters would pose for each other. Chapter 1 detailing the early life of the mathematically precocious includes a section on mathematical competitions used to provide an entree into mathematical life for many students who otherwise would not have had any contact with mathematics beyond the trivialization of the subject available in most school settings. Clearly competitive mathematics has been an important spur to many leading mathematicians, for example, to Gábor Szegö who eventually became chair of the mathematics department at Standford University. As he recounted his high school years in Hungary from 1910 -1912, "'I would wait eagerly for the arrival of the monthly issue [of the Hungarian high school math newspaper] and my first concern was to look at the problems without delay'" (p. 30).

There is a danger, however, that mathematics itself becomes distorted through an overemphasis on competition. The British tradition of mathematics training for hundreds of years was oriented by Cambridge University's Tripos Examinations whose questions "were usually of considerable mechanical difficulty–but unfortunately did not give any opportunity for the candidate to show mathematical imagination or insight or any quality that a creative mathematician needs" (Snow, 1967/1992, p. 22). For some individuals, the extrinsic rewards of prizes and fame can come to supersede the solidarity of mutual appreciation that must be at the heart of mathematics. In Chapter 2, Hersh and John-Steiner review the controversy over the recent proof of the Poincaré Conjecture in which the Chinese mathematician S.-T. Yau "encouraged two of his younger colleagues ... to write up [and publish] their own extended proof" (p. 71) of a result that already had been sketched by Russian mathematician, G. Perelman. In protest (and presumably, discouragement) Perelman refused the Fields Medal (the equivalent of the Nobel Prize for Mathematics) offered to him for this work, and may have retreated to some extent from active mathematical life. Though Yau may have been motivated by national pride, rather than personal advantage, the harm that can come from elevation of competition over mutual collaboration is apparent.

In refusing the Fields Medal, Perelman explained, "'Everybody understood that if the proof is correct then no other recognition is needed'" (p. 72), which Hersh and John-Steiner interpret as "a beautiful example of intrinsic scientific motivation" (p. 73). Intrinsic joys of mathematical creation permeate the book. Yet, as recounted in Chapter 4, these joys are counterbalanced by some unique pressures. A peculiarity of mathematics, in contrast with all other fields of scholarship, is the historical continuity that makes problems of decades ago, even centuries ago, still current. This continuity is part of what establishes the sense of eternalness and perfection of mathematics. But the flip side is that standards for comparison of one's contributions are not restricted to the current era. One also is in competition with the greatest minds of the past. As a result, "mathematicians are particularly vulnerable to a sense of inadequacy in a profession that remembers and honors so many of its most illustrious contributors" (p. 335).

Hersh and John-Steiner paint a fascinating picture of a complex psychology of mathematical work bounded internally by the joy and frustration of deep theory and hard problems, and externally by the discipline-internal dynamic of humility contrasted with the enormous awe in which mathematical intellect is held by the general public. The internal and external demands experienced by mathematicians can become debilitating: "we all know that our results, our publications, are supposed to be completely correct, logically irrefutable. We also know that in very many cases, even though everything looks right, there remains an aching uncertainty. ... What if it turns out to be all wrong? ... then disaster and disgrace would be utter and final. All this is in fact delusional, yet it is somehow part of the ethos that we absorb somewhere along the line in our training and indoctrination" (p. 127). These emotional dynamics are pursued by the authors into what, fortunately, are infrequent cases of actual madness and violence, in one case diagnosed by psychiatrist Henri Baruk as "'morbid rationalism ... a crime of logic, performed in the name of absolute rationalism as dangerous as any spontaneous passion'" (p. 133). They conclude this theme with a sense of humanism that permeates the book.

> How dangerous this dogma [of mathematical infallibility] can be! Logic can never be anything but a tool, an action, or a procedure carried out by a *human being* .... Logic, such an essential tool of science and philosophy, sometimes becomes a sort of false god, outranking the most fundamental human impulses, such as "thou shalt not kill." (p. 135)

The scholarly excellence of *Loving + Hating Mathematics* filters through in its non-academic tone. Hersh and John-Steiner have profound things to say about mathematical culture. Yet for the most part what they do is speak through the eloquent and passionate words of the scores of mathematicians, educators, and psychologists whom they quote and reference. The back matter for the book includes a generous annotated list of books consulted by the authors, and a separate section of biographical sketches of mathematicians cited. It is the privilege of being so in touch with the living history of mathematics that enchants.

Snow, C. P. (1967/1992). Forward. In G. H. Hardy, *A mathematician's apology* (pp. 9-58). New York: Cambridge University Press.