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PSYCHOLOGICAL CONDITIONS OF THE ORIGIN OF IDEAL* ACTS

A major tenet of Vygotskian Soviet psychology is that higher mental functions represent culturally, historically accumulated knowledge that is transmitted to the child in the course of his/her socialization. This idea of the gradual internalization of initially external (to the individual) modes of thinking is, of course, not unique to Soviet psychology.

Wherever it is encountered, an internalization theory faces the difficult task of specifying how, in detail, the internalization process takes place. The article that follows spells out the process of internalization of mathematical thinking involving addition to an unusually precise level. Davydov consistently rejects the notion that

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*The term ideal [idealnyi] is used by the authors here in connection with a process or action that is carried out on the plane of ideas as opposed to a plane of external, practical behavior. "Ideal" in this sense contrasts with "material" and does not mean optimal or perfect. — Ed.

to be able to talk about a concept or to fulfill it under a restricted set of circumstances is not enough to indicate that it has been thoroughly internalized; rather, the individual must demonstrate an ability to carry out the requisite operations in a variety of appropriate modalities.

An especially interesting point in this article is the evidence that certain modalities (gestures, for example) may inherently be particularly bound to certain stimulus configurations, thus reducing the generalizability of the associated cognitive operations.

Details on the process of internalization are rare in the relevant literature. Davydov and Andronov's article is an especially clear model of how such research can be carried out in meticulous detail.

Michael Cole, Editor

Purpose of Our Investigations

Years of research on the fundamental psychological problem of the transition from successive to simultaneous perceptual and intellectual processes have invariably produced descriptions stressing the ways developmentally primary and expanded forms of cognition are transformed into new, abbreviated, and more complicated forms. From the multitude of studies that deal with this problem, we shall focus on a few authors' work that is directly connected with this phenomenon of abbreviation (see, for example, references 8, 9, 21, 22, 23, 25, 26, among others).

Psychologists who have studied the step-by-step development of cognitive acts have been especially interested in these phenomena (see references 2, 15, 16, 18, 19, 24, among others). In explaining the empirical studies in this area and discussing the problem of the psychological nature of abbreviated mental acts, P. Ya. Galperin has written:

> Abbreviated operations do not simply disappear. They take on a status in which they are treated as if they had been performed and are hence being "kept in mind." As a result, action acquires a very specific form... At both the perceptual and the cognitive levels, the object content of an action is no longer carried out, but is only "being kept in mind" beyond the limits of what is actually being done. The real action always takes the form of a motion going directly from a starting point to an end point, ignoring the objective interrelations of the problem, as if indicating that real action is distinct from object action while neglecting its objective logic and difficulties. (Pp. 253–54)

In analyzing the psychological problems that follow from this important thesis, we must distinguish a few of the principal ideas. First, we must distinguish the object content of an act* — its objective logic, which corresponds to the objective circumstances of the task at hand — from the actual, abbreviated accomplishment of this act, represented by a kind of movement that contradicts this logic and is incompatible with it. Abbreviated object operations that are beyond the bounds of what is actually being done are operations "only in the mind," and are viewed as if they were carried out. Such implicitly understood operations are carried out in the form of ideal or cognitive "movements" that differ in their logic and possibilities from movements of the primary, material act. The existence of this implicit form reflects the rational basis of human problem-solving (see, for example, reference 1, p. 274).

Psychologists have now accumulated considerable data that enable them to describe in detail the external features of abbreviated (ideal, cognitive) acts in contrast to their material, ob-

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*The Russian term predmetnoe soderzhanie refers to the contents of the activity that are being represented psychologically. Similarly, object operations refer to mental operations based on or using objects. — Ed.

ject prototypes. We think, however, that a number of substantial problems still remain unsolved, problems having to do with the internal conditions and laws of the process of the emergence of ideal acts. Central among these problems are the following:

1. What is the general logical-psychological method for discerning the object content of acts corresponding to the objective relations of the tasks at hand?

2. What are the foundations and the criteria for discriminating between object acts carried out in accordance with the requirements of their objective logic and the various forms (levels) of abbreviated "actual" acts whose logic contrasts with this logic? (1)

3. What is the psychological content of specific human acts in the transition from material, object acts that are actually carried out into implicitly understood (as if already completed) operations of ideal acts?

4. What are the psychological and psychophysiological forms for carrying out the ideal movements (processes) representing abbreviated (implicit) operations constituting internal human cognitive acts?

5. What are the general principles for constructing and employing a method for the psychological study of internalization processes in light of the concepts outlined above?

In our view, the contemporary theory of internalization will soon be occupied with all these interrelated problems. As Gal'perin has noted, psychologists are "faced with the task of studying the laws of abbreviation and the truly enormous possibilities that abbreviated cognitive acts open up for thinking" [1. P. 275].

In the process of resolving this task, however, a serious difficulty has arisen in connection with examination of the specific content with which the main levels or forms of abbreviated acts are combined. These forms have now been described more or less in detail; but a genuine specification of cognitive acts, their "logic," and means for carrying them out should be determined not so much in terms of the external forms of abbreviated acts (for example, in terms of their execution in the absence of material objects and "to oneself") as by means of carrying out the initial task itself. That is, we must study internalization by means of the substantive method, which is essentially different from conditions in which the task is carried out by material, object acts.

To be sure, in the psychology of perception and cognition much has been described that indicates how perceptual acts in human beings are abbreviated when the most informative content of an object is distinguished with the aid of new means. But the demonstration of this finding has not been accompanied by any substantive analysis of the cognitive means intrinsically linked with the specific content of objects, with their concrete object or general, logical characteristics (e.g., with the geometric features of what is perceived, etc.).

In other words, in addition to a description of the basic levels or forms of executing an act (material, spoken, external speech "to oneself," or real internal speech), it is necessary to ascertain the new cognitive methods associated with them in order to determine the reasons for the shift from one mode to another. Essentially, this specification of mechanisms of change requires a special method of psychological analysis of the processes of the genesis and transformation of the modes by which human beings carry out an act as part of a unified process together with changes in their forms (levels). This general method has not yet been created in Soviet psychology, although without it any modern theory of internalization of acts is impossible. We should point out that Piaget's followers have moved in a similar direction, but of course on the basis of their own fundamental concept (Piaget's school attaches special significance to the logical aspects of internalization).

The development of such a general method presumes careful study of the processes of internalization of concrete acts relating to specified areas of knowledge (mathematical, linguistic, historical, etc.). In the opinion of certain specialists, this has, in a sense, diverted the attention of psychologists from dealing with their own "special" and independent problems.

But our experience in studying the processes by which certain mathematical actions are formed (for example, measurement, counting, multiplication, etc. [3, 5, 6]) has shown that without such psychological groundwork, a general theory of internalization cannot be created.

Quite early in our studies of the processes of step-by-step development of mental acts we discovered, using mathematical addition as an example, that the method employed in a cognitive act is characterized by the possibilities of operating with objects, first, unrelated to their immediate material expression and, second, unrelated to the initial form of their transformation [4]. It was shown that the unique form of an abbreviated mental act could be observed when its objects were represented in some directly material object form — i.e., its expansion in this case took place completely differently from the accomplishment of a genuinely material act [3, 15].

At this level we thought it expedient to return, after a long interruption, to an examination of the problems of internalization as illustrated by the operation of addition in preschool-age children. It was necessary once again to verify previously accumulated experience and to specify and expand this experience on the basis of contemporary methods and means for collecting empirical data. (The use of a video recorder to gather new data was quite important in our experiments.) Below we shall present empirical data combining the results of previous experiments and of new experiments carried out in the past few years. (2)

A description of the data on the development of the act of addition by children is given so that in the subsequent discussion we can clearly show the relationship between the abbreviation of an act and the hallmarks of its cognitive form. We shall also describe certain psychological conditions of the genesis of abbreviated ideal "cognitive" acts.

Experimental Data

Kindergarten children between the ages of four and seven were given a set of addition problems. In each of these problems the first term was designated verbally. This term was varied by concretely instantiating the corresponding quantitative set. The second term was left in object form and was not verbally designated by a single word (this required it to be counted one by one, since it was not possible to respond with a ready-made formula reproduced from memory; this was true of all the problems except the first, purely verbal task). The addition tasks were given only to children who in a preliminary test had demonstrated the ability to count from one or from any number (forward and backward) and who indicated that they had knowledge of preceding and succeeding numbers, knowledge of written numbers, and an understanding of the meaning of the action of addition, which they demonstrated by adding together objects, both by bringing them together and joining them with or without a material act.

The tasks were as follows:

(1) Purely verbal: "Add three to four. How many do they make together?"

(2) A verbal task in which designation of the first term was accompanied by a pointing gesture in the direction of a certain place on the table: "Here are five (pointing gesture), and you have to add this many (pointing gesture to objects without naming their quantity). How many do they make together?"

(3) A task in which the first term is given in the form of [written] digits: "Here is this many (a digit is shown), and you have to add this many (objects). How many is that altogether?"

(4) A task in which the first term is given in the form of a group of objects enclosed in a box: "Here are six (pointing to the box), and you have to add this many (objects). How many is that altogether?"

(5) A task in which the first term is also presented in the form of a haphazardly arranged (open) set of objects: "Here are four (the objects are put down), and you have to add this many (objects). How many is that altogether?"

In both the test and in the training trials with the children, the following aspects of behavior were noted:

(1) the means by which the child did the addition (enumeration, addition);

(2) the form of the objects being added (open or enclosed objects, digits, purely verbal tasks, or verbal tasks accompanied by a gesture, etc.);

(3) the nature of hand and finger movements (tapping with the finger on the table, "feeling" the objects enclosed in the box, a continuous movement of the hand along a row of objects, etc.);

(4) special characteristics of the way the numbers were enumerated (rapid and even, with pauses and reversals, accented, "drawn out," etc.).

The data from the study enabled us to distinguish three main levels of performing addition, for each of which a specific relationship between the form in which quantity was expressed and the method of performing the act was characteristic.

The first level consisted of summing up the items in a set of objects by means of one-by-one enumeration, the child's finger touching each object in turn (in the study done in 1977–1979, this level was observed in 30 children of the 220 we studied). This enumeration of all the elements in a set is dictated by the fact that the set is represented in object form. (3) Some of the subjects duplicated with hand movements the external form of the objects to be added (for example, sticks or circles).

At the second level, the task was typically performed without an object-based first term. This was done by means of a verbal enumeration of the terms one by one, starting with the number one (44 children). For example, the problem "4 add 2" was done as follows: "1, 2, 3, 4 – 5, 6, makes 6." Such enumeration was usually accompanied by movements of the hands, fingers, head, and body. These movements took several forms: a sweeping movement of the finger along the table, pressing or tapping at some point on the table, a series of nods of the head (from distinct to barely perceptible). The more fully the first term was rendered in object form (tasks 2–4), the more expanded were the movements (prohibiting these movements usually interfered with counting).

To determine the function of these movements, it was necessary to examine the conditions of their occurrence in the children as they switched from adding objects together to adding sets. The first form of addition was not given explicitly in object form. This was done in a formative (training) experiment. To one seven-year-old who was able to add together only open sets of objects (first level), we gave tasks in which the objects were enclosed in a box. At first the experimenter stressed the designation of the first term by uttering it and, opening the box, showed the child the objects in it. The box was then closed again, and the task was rephrased: "There are five in the box and you've got to add two more." After a few variations of this task, all the children began to move their fingers over the box and orally to enumerate the elements of the first term successively (one, two, . . .), after which they passed on to the second term (the open objects). In the next stage the tasks involved an empty box, digits, and gestures. All the subjects managed to perform these tasks successfully.

Our observations on the characteristic features of these children's behavior indicated that the finger movements on the box served as a means of "resurrecting" the sets of objects enclosed in the box and, later, sets not present at all while, at the same time, employing these movements as the objects to be added together. Forced to operate with a quantity not explicitly given, the children began to use the same movements they had developed in operating with the objects themselves (earlier the children had touched the objects with their fingers as they named the numbers). Each movement (squeezing, pressing, tapping) was now a substitute (representation) of the immediate material item, which returned only in the role of an object of an act.

It is interesting that to the extent that a child switched from an act with objects exposed for counting to an act with a hypothetical set, the movements themselves changed from slow, extended pressures as the finger moved along to light, quick taps at one place on the table. In the transition to a purely verbal task, the movements were increasingly reduced (compacted), although they did not disappear completely, and finally were replaced by whispering "to oneself" (this was revealed by movement of the Adam's apple).

All these movements were still a manifestation of a method for the material act of addition — one-by-one counting aloud, in a whisper, or to oneself.

At the third level of internalization, addition was done by joining the elements of the second term to the objects constituting the first term taken as a whole. The subjects on this level (50 children) did the addition tasks as follows: They made a characteristic, continuous (sweeping) wave of the hand over all the objects in the first term as they drawled the first cardinal number and then continued by adding to it the elements of the second term ("s-i-i-x, seven, eight"). This continuous movement enabled them to operate with all the units of the term without distinguishing them individually. Counting on the basis of this continuous movement over all of the objects in the first term was a stage beyond the method of addition characteristic of the material level of an act: this new method indicated the emergence of a genuinely mental act of addition.

During our studies we discovered a group of subjects who also did addition of objects by joining one group to another but did not perform the continuous movement with the finger over the first set of objects. These subjects pointed with a finger to only one element in the set and uttered the cardinal numeral (without any intonational emphasis) and then went on to enumerate the second term ("four — five, six"). When the experimenter asked, "Is this really four? This is only one, and to four we have to add two," some of the children (30 subjects) responded by making the continuous hand movement over all the objects in the first term and drawling out the first cardinal number and then adding the second term. Other subjects (40 children) responded to the question by shifting a finger to the next object in the first term and again giving the cardinal name that had been given to them. The experimenter repeated the question. The child would switch his finger to the third object in this term, etc. Prohibition of this technique resulted in the child's refusing to do the task or in his switching to one-by-one counting. It was characteristic in this case that when the experimenter spoke to these subjects outside the adding situation with a request, for example, "Give me four," they would always apply the cardinal name to all the objects in a group. In the adding exercises, however, these children again designated only separate elements in the term as "four."

Some of the children combined counting and combining in a unique way: the child touched one of the objects in a set with his finger, designated it with the cardinal number corresponding to the number of objects in the first term, but instead of going on to the second term, continued to count the elements in the first term, after which he went on to the second term. For example, the exercise "add three to four" was done as follows: "four – five, six, seven – eight, nine, ten."

The subjects of this second group initially formally exhibited adding as their method of summing up the objects. But as a special study showed, this "adding" consisted in the ability to immediately "count beyond" the indicated cardinal number. This was manifested in particular by the fact that uttering the cardinal numbers during formal adding had the same intonational characteristics as in "simple" counting (in real adding, the cardinal number designating the first term was drawled to a certain extent, with a special intonation). This formal kind of adding we dubbed simulated.

We also encountered simulated adding in other situations. For example, in studying the characteristics of adding abstract sets, we found a group of children (17 subjects) who used addition when the task was presented with digits, but performed the other tasks (with a gesture, with explicit objects, etc.) by counting. These subjects were given a special task in which a digit alternated with an explicit object. As soon as the child began to count the objects in the first term, the experimenter would place a digit on them. The subject would then quickly and emphatically alter his method of summing and go over to combining. If the digit was taken away, the subject would again switch to counting. Hence, these subjects also were unable to relate the cardinal number to the whole set of objects. This was simulated addition in the form of "counting further."

To determine how stable these methods of addition were, the subjects who were able to do simulated addition and real addition were given special tasks in one of which the first term without objects was designated by the cardinal "million" and in the other, imaginary words designating a specific number of objects — for example, AR (1), UR (2), ER (3), etc., were used as the cardinal numbers.

It was found that all the subjects who did real addition did not alter their method of addition in either of the two tasks. They performed their continuous sweeping hand movements over the table (in the "million" exercise) or over the objects in the first term (in the exercise with imaginary words). All the children at the same time drawled the first cardinal number and then went on to the second term.

The subjects who did simulated adding also did not alter their method of addition in the "million" exercise. They would touch the table with a finger and quickly, as in simple counting, utter the cardinal "million," and then go on to adding the second term (for example, "million, million and one, million and two, million and three," etc.). But in doing the exercise with the imaginary words, these subjects fell into two groups: one group (18 children) did it only by enumerating; the other (22 children) continued to add, but again only in the simulated form. These children would touch one of the objects in the first term with a finger and utter the cardinal name (quickly and evenly, as in simple counting) and then go on to the second term. But it the experimenter asked, "Is this really ER? This is AR, and we have to add this much to ER," then the subjects would point to another object in the same term, then to a third, or shift to enumerating.

In the next stage of the study we determined the characteristics of general intellectual development in the children who were able to do both simulated and real addition. These studies were based on the following considerations. We assumed that it would be difficult to show the inadequacies of simulated addition as a mathematical act by studying directly observable properties of adding and subtracting numbers. Indeed, summation in the form of simulated addition brings a child to the same outward result as real addition (simulated addition does not distort the formal result of summation). Hence, the properties of simulated addition, compared with real addition, were conveniently correlated with the characteristics of certain general features of a child's intellectual development. We assumed that children who did real addition would manifest a higher level of intellectual development.

One of the most important indicators of a rather high level of cognitive development, in our view, is the presence of reflection as a special ability to examine the reasons underlying the methods of one's own acts. (4) Reflection, as an aspect of thought, is internally linked with the capacity of human beings to resolve contradictions. Therefore, it was natural to assume that if a child were already able to resolve certain contradictions, then he probably had achieved a certain level of reflection; if he had not achieved this level of reflection, then in all likelihood he would not be able to resolve contradictions.

For our subjects, the encounter with contradiction took place in a situation in which the experimenter asked, "Is this really five, six, seven, etc.)? No, it's 1!" when the child ascribed some cardinal number to the entire group and, at the same time, to one of its elements. As we have shown above, for some of the subjects this question was not required at all, since they had already discovered a method for resolving the particular contradiction on their own, such as their continuous hand movement, or shift of gaze, over all the elements in the first term, or a special, accented pronunciation of the cardinal number designating the entire set of objects.

In another group of subjects, who designated only one element in a set with a cardinal name, this question was able to demonstrate clearly the existence of a method for resolving such contradictions. A third of the children, however, were unable to resolve this contradiction at all (after the experimenter asked his question, the children again pointed to only one element in the first term or refused to do the task). Simulated addition was characteristic of these children.

To be sure that our reasoning above was correct, we conducted a special study to determine the presence of reflection in each child. The subjects in these experiments were children who could do simulated and real addition but could carry out addition of abstract sets only by enumeration. We used a procedure developed by A. Z. Zak [7], which we modified for the age of these children. The results are given in the table.

Modes of adding	Total number of subjects	Number of subjects with reflection	
		Number	%
Real addition	80	72	90
Simulated addition	40	3	7.5
Enumeration	44	6	13.6

Most of the children who did summation by means of real addition also possessed reflection. In addition, the percentage of children who had reflection was very low among those who did simulation or enumeration. Thus, the presence of real addition in children is related in some way or another to a higher level of cognitive development, particularly to the ability to reflect.

The next task in our investigation involved an experimental study of the conditions of formation of real addition in children. First, we developed the ability to carry out real addition in five children who were able to engage in stable verbal counting (the second level of addition).

The experimenter gave the subjects a task by using a gesture or a number. When the child began to count the first term, the experimenter interrupted with questions and special instructions: "How many is this (pointing to the number or to a place on the table earlier indicated with a gesture)? Four?

So then add two to four." After several such instructions, all the subjects shifted to addition: they would touch the number or the indicated place on the table with a finger, pronounce the particular cardinal number designating the first term, and add to it the elements of the second term. All other tasks with nonobject first terms were done in the same way.

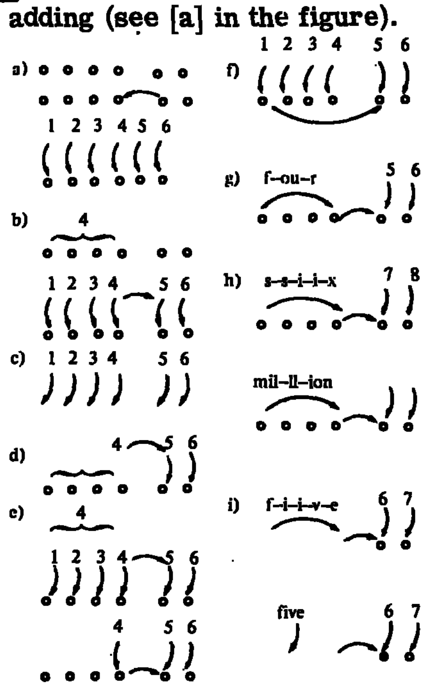
Then the children were given an object task. This was again done only by enumeration. When the experimenter prohibited this method of summation, some of the subjects switched to adding in the form of relating the particular cardinal number to one element in the first term. When the children encountered a contradiction of the type "Is this really four? No, it's one," they were unable to resolve it. Another group of children refused to perform the task at all. Thus, the adding we developed in this way proved to be simulated, with the properties we have described above.

Real adding was developed in 20 children in the process of summing sets of objects. These children were able to add sets of objects and abstract sets by enumeration quite well. The experimental technique involved the following: The subjects were asked, "How many are here?" just at the moment when they were getting ready to place a hand on the last element in the first term, which had been moved some distance away from the other objects. Under these conditions, the continuation of the movement along the row coincided with the child's naming the cardinal number designating the entire group of the first term, after which the child would go on to the second term. After two or three such tasks, the experimenter asked, "How many is this?" after two, and then three, etc., elements until the objects in the first term were completely enumerated. The child had to combine his reply with a continuing movement over the elements in the set.

In all these 20 children, real addition was developed. It was typical that the number linked with the movement was pronounced in a drawl. Addition originally developed only for explicit sets of objects later was easily transferred to other forms of the task (tasks with an empty box, with a digit, and with a gesture). As the task with nonobject terms was completed, the scope of the continuous movement gradually diminished, and the drawl in uttering the cardinal numbers became less and less noticeable.

Discussion of Results

These findings enable us to describe a general scheme for internalization of the mathematical act of addition in children. After outlining it, we shall attempt to explain it psychologically. Initially an adult gives the child a task in addition, presenting him with two sets of objects (terms) and requiring him to determine the number referring to the total group (sum). The child's action consists of physically bringing together (combining) the different sets (again, the adult shows the child how to do this). But in this combination, each set loses its ordinal definition, and the child can obtain the result of addition only by one-by-one enumeration of the elements in the sum. This is the initial material act of adding (see [a] in the figure).



An important aspect of the internalization of this act is the child's mastery of the ability to determine the sum without bringing together the sets of objects. An interesting situation arises: the child must find a sum when he already knows the cardinal numbers corresponding to the terms. On the basis of this knowledge he is able to add the elements of the second term to the first term (in principle, he can use an addition table to find the sum). This, however, does not take place: to obtain the sum, the child enumerates one by one the items in the first term, even though their sum is known to him ([b] in the figure).

The paradox here consists in the fact that the child, on the one hand, is already able to count from any number and, moreover, outside an adding situation distinctly relates a cardinal number to the entire set of objects. But in the act of addition, this "ability" and "knowledge" do not guarantee that the child will consider any set of objects as an integral term. Perhaps the child can relate cardinal numbers only to material groups of objects? But observations show that even when "purely" verbal terms are given, addition still takes place by means of enumeration (in this case it is based on reduced hand movements, "restoring" the object units – [c] in the figure). It is striking that some of the children really do take into account the possibility of adding when a first term is given verbally; these children use their ability immediately to "count further" from the given cardinal number! Formally this appears to be addition ([d] in the figure); but when an adult required that a cardinal number be correlated with a group of objects, the children would switch to counting or would apply the cardinal number to only one element in the term ([e] in the figure).

How can a child's paradoxical counting of a term already known to him and the fact of simulated addition be explained psychologically? For an explanation we need, in our opinion, a special analysis of the content that represents the object of action for the child. Addition involves counting from any number in a form that is based on its ordinal characteristic. The child must also take into account the word itself – the cardinal name – which has a meaning unrelated to the set of objects (in addition that does not bring the concrete items of the term together, their direct [cardinal] expression is unnecessary and superfluous). But the cardinal name is a resultative quantitative aspect expressed through its relationship to all objects in the set. An adult very early teaches a child to take this aspect of the set into account. If, on the other hand, the object component is brought together physically with its verbal cardinal definition, the child, in some form or another, senses their incompatibility, a clash and a contradiction between them (essentially this is a contradiction between the quantitative and the ordinal aspects of a number). (5)

Hence, in one case the child will ignore the ordinal definition of the cardinal word and will have to recount an already defined term. In another case he will ignore the quantitative definition of the first object in the term and will "apparently" add the elements of the second term to the cardinal word corresponding only to one element in the first component (this aspect is, in fact, brought out by the experimenter's question "Is this really four? No, it's one!"). We observed a characteristic fact that demonstrated the tendency of some children to take into account simultaneously the ordinal and the quantitative aspects of a number in some hybrid form. After naming one element with the given cardinal name, the children then counted from it all the objects in the first term (for example, in the exercise "4 + 2," this appears as follows: "4, 5, 6, 7, 8, 9 will be 9," [f] in the figure).

This contradiction, which shows up in the child's own activity, can, we think, be resolved by him only by changing the way he deals with the terms represented by objects, continuing the actual movement of his hand over all the objects; he does not stop on each element, but immediately names the result familiar to him – the cardinal number – and then goes from it to the second term (this kind of addition involves taking into account the ordinal aspect). The continuous sweeping movement of the child's hand over a series of items as he drawls out the cardinal number also satisfies the requirement of simultaneously taking into account both aspects of the number – this movement brings out the real unity in the child's own activity ([g] in the figure).

In this last case, one physical group of objects may be designated by the child by different numbers, i.e., by any number, since it is indicated to him beforehand and there is no need to point out all the elements of this group. Hence, when the experimenter designated one and the same group of objects with different numbers (up to a million, of which, of course, the child has no precise conception), the child had a means of relating to it as the representative of any possible number; this means was the sweeping movement of the child's hand, which designated the concrete number given to him ([h] in the figure).

In this situation it is not the number that represents the corresponding set of objects; on the contrary, a certain set represents any number. In this inversion of the relationship between the group of objects and the number we find the secret of addition as a means of summation, the secret of the process by which the child himself discovers the unity and coincidence of the quantitative and ordinal aspects of a number – a discovery he makes through the continuous movement of his hand over the row of objects in the first term. It may be said that through this movement emerges the genuine "cognitive plane" of performance of the act of addition, since only by means of such a movement does the group of objects begin to function as an integral structure, as a term, for the child. Then the act of addition is also formed in the child, in contrast to the act of enumerating with which he had solved addition problems earlier. Now the child has to accomplish this task by means of an act adequate to it, but that act has to become cognitive in the way it expresses the relationship between the group of objects and the number. For a practical act of addition, numbers are typically used as representatives (substitutes) of certain groups of objects, whereas for a cognitive act of addition, the group of objects is used as a symbolic representation of numbers. The transition from enumeration to addition on the basis of a sweeping movement of the hand over the group of objects also brings about a change in their interrelation, namely, a shift to the kind of relation that is inherent in a cognitive act.

It is clear that, in explaining the secret of addition in this way, the process of its formation in both the child's spontaneous and independent activity and in a specific instructional process presupposes some reference to objectively represented terms: a change in the functional relation between a set of objects and a number is possible only when some reference point is present during the transition to a new method of effecting this relation. This is why when a cardinal number is divorced from the group of objects (for example, when the first term is introduced in the form of digits), when the child does not develop this new method, we observe a phenomenon of simulated addition (the act of counting is not transformed into a cognitive act of summation). The child's switch to this fundamentally new method, which is especially characteristic of a cognitive act, makes the "scope" of his sweeping movement and the "magnitude" of the number a matter of indifference to him: when the scales are changed, any such movement related to any number expresses the new relation between the set of objects and numbers. As a result, the child no longer needs a term consisting of objects in his addition, and the movement is quickly reduced; and his peculiar intonation in pronouncing the cardinal number begins to symbolize its use as a term (this symbolization here functions as one of the essential aspects of internalization ([i] in the figure).

Our explanation of this problem followed the course of argument concerning the conditions and causes of changes in the content and form of an act as it is internalized. At this level our answer to the question concerning the criterion of a cognitive act in the strict sense, in contrast to its corresponding material act, becomes intelligible: the difference between the two is related to a change in the way it is performed, which is based on a different relation between the material-objective and symbolic-verbal means for fixating their respective objects. Without a special and careful examination of this psychological reality, one cannot really speak of an adequate method for studying the processes of internalization.

Our research has shown that, in simulated addition, children display no reflection; in contrast, in real addition they do reflect. These findings may be explained if one assumes that the sensing of a contradiction and the search for a way to resolve it presume a special ability to examine the foundations of one's own acts and to analyze the possible conditions for their effective execution, i.e., reflection. Hence, we can justify seeing its essential psychological role as residing in the emergence of real addition in children as a means of resolving this contradiction between the quantitative and the ordinal aspects of numbers, which the child discovers in his own acts as he attempts to carry out summation.

A detailed analysis of the conditions of the genesis of addition as one of the concrete ways human beings perform mathematical summation is warranted if it will help to resolve the central psychological problems concerning the occurrence of abbreviated ideal, cognitive acts. What light do the results of our study shed on these questions? Since they were obtained in a "test tube," so to speak, they to a certain extent give a concrete form to the sense of those questions concerning the theory of internalization that were formulated at the beginning of our paper. (6)

First it is important to distinguish between the logic of material acts and the logic of ideal acts. We have demonstrated the various possibilities and psychological characteristics of these different forms of logic using a mathematical act as an example. The creation of an integrated theory for them should be based on the principle of dialectical logic, which, as Il'enkov says, "is not only a universal schema for subjective activity but also, at the same time, a universal schema for changing any natural and social historical material on which this activity is carried out and which always places objective requirements on it" [10. P. 5].

The logical psychological method for studying the emergence of ideal acts can only be the content-oriented [soderzhatel'noe]-developmental method. (7) It should constantly draw substantive, concrete material from that domain of reality that is transformed by the material acts associated with its objective needs (for example, without drawing on mathematical knowledge concerning the act of addition it is impossible to understand changes in the way addition is carried out).

Our description of the development of mathematical cognitive acts has a distinctly developmental aspect. It bears not only on the origins of particular acts but also on the genesis of thought in general, which functions as an "ideal component of real activity of social man" [10. P. 5]. (8)

The transition from material acts to ideal acts in human beings is intimately related to the use of the means of symbolization. In our example of the act of addition this was manifested quite clearly; it was only by transforming a set of objects into the symbol of a number that the child was able to resolve the contradiction he encountered in the process of enumeration. Thanks to this transformation the ideal act of addition was formed. The existence of a symbol opens tremendous possibilities to the child in the use of numbers in the logic of ideal acts.

The essence of a symbol has been brilliantly expressed by A. F. Losev: "...the essence of a symbol comprises what is never directly given by a thing or by reality, but rather what is presupposed by it, not the thing itself or reality, as constructed, but its generating principle, not its supposition, but its presupposition, its assumption" [13. P. 12]. Further: "As regards a symbol of a thing, it contains in implicit form all possible manifestations of that thing" [13. P. 17].

The logic of ideal acts as acts with symbols belongs to that domain in which the ideal is "nothing but the form of a thing outside that thing" [10. P. 189]. From this point of view, the ideal exists only in a recurrent cyclical movement, i.e., "thing – act – word – act – thing" [10. P. 193]. The emergence of the symbol in a child in the addition of numbers shows that the preschool child can "sense" a contradiction in his own acts and independently find an adequate means of resolving such a contradiction. This indicates the deeply dialectical essence of the thought of a child.

In our example an ideal act of addition, based on an objective symbol, takes place in the form of a real motor act (the sweeping movement of the hand as the cardinal number is pronounced in a drawl). Then, in a compact, reduced form, this movement itself becomes the symbol of the number, manifested in unique gestures and in the accented pronunciation of the cardinal numbers. A study of the emergence and transformation of this movement as a basic component of an ideal act could be a key problem for contempory psychophysiology as it studies processes of internalization (in particular, the interesting question of the further fate of this reduced movement).

This problem is of fundamental importance. Thus, in our example, initially the symbol was the hand's movements together with articulation, and subsequently only the reduced articulation as a basic component of the word in designating a number. This aspect should be kept in mind in questions concerning complex symbols and their role in ideal acts. "The directly ideal takes place in symbols and through symbols, i.e., through an external sensorily perceptible visible or audible body of a word" (Il'enkov [10. P. 193]).

An analysis of our "test tube" example touches only cursorily on the way the underlying complex, logical, psychological problems are resolved. Extensive empirical material is necessary for a more comprehensive explanation. This material can be gathered by using those general logical psychological ideas that relate to the nature of the ideal.

Notes

1) Here it is extremely important to determine the essence and chief characteristics of the "objective logic" of the object content of acts and other, different types of logic corresponding to abbreviated ideal acts.

2) We have used, in particular, materials obtained by the student graduates L. A. Khristich and V. Ya. Dobrokhotova

(Moscow State Correspondence Teachers' Institute).

3) This level, which is defined in terms of the relationship between forms of expression, quantity, and ways of addition, is preceded by another two stages in which this act is formed: the child's acceptance of the addition exercise and the act of counting after the objects are gathered together (we do not examine these stages in this article).

4) A. Z. Zak [7], L. K. Maksimov [14], and V. T. Nosatov [17] have made a detailed psychological study of reflection as one of the main components of thought.

5) Piaget has described a unique contradiction between the quantitative and the ordinal aspects of number and the psychological difficulties the child experiences in employing this concept [20. Pp. 386–90].

6) The results of this study are internally linked with the findings of our other investigations carried out on different objects; they have served as the foundation for a unified theoretical position [5, 6, etc.].

7) G. P. Shchedrovitskii (see [27] and others) made an important contribution to the problem of a content-oriented, developmental method in logic and psychology.

8) A. N. Leont'ev [12], S. L. Rubinshtein [21], and others have developed this idea of the emergence of thought in psychology.

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