CUSP – Real-world Science Workshop VI: Introduction to Waves 10/31/02

Homework Set:

1. During this session we discussed the basics of wave phenomena and came up with the most basic features of what it is to behave like a wave. However, we casually call a lot of things "waves" which do not have the same features as these most basic features and so may not really behave like the waves we discussed. Consider the exhibits in the Exploratorium that seem to involve waves and determine which ones actually do involve waves as discussed in our session. For phenomena that, strictly speaking, are not waves give one reason why not. (e.g. A stadium wave has wavelike characteristics in that the medium does not experience any net motion, but there really is no true restorative force causing the wave propagation… a person could stand up quickly and no wave would necessarily propagate) Note: The tall spinning sheet/umbrella sheet is a tough one, but take a shot at it… in the end, whether or not something is a wave depends on whether or not we can successfully apply the analytical tools which we know work on waves.

2. Draw an atomistic picture sound waves of one frequency, at one given time, travelling along a "line" of air. Below the picture draw the way this wave would be represented on a standard "side-view" or Displacement-vs.-x graph. Repeat this for the same frequency, but larger amplitude. (Feel free to draw the atomistic pictures in as minimalist a style as you wish)

3. Show that both the Sine and Cosine functions are appropriate solutions to the Wave Equation. Why do most scientists change the argument in "Sin(argument)" or "Cos(argument)" from argument=(x-vt) or (x+vt) to be argument=$(2\pi/\lambda)$(x-vt) or $(2\pi/\lambda)$(x+vt) ? If we define $\omega=2\pi\nu$ and $k=2\pi/\lambda$, what does the argument look like, in terms of x,t,ω and k only? (ω is called angular frequency, k the wavenumber and ν, λ frequency and wavelength as usual) What's the effect of adding some dimensionless (or radians) value ϕ to the new argument of Sine or Cosine? This value is often referred to as the phase and, along with the other terms defined here allows for a different way to examine and represent the same wave.
(**http://id.mind.net/~zona/mstm/physics/waves/waveAdder/WaveAdder1.html**
lets you vary the phase to observe what changing the phase does)
We'll save most of this for later, but for now what would it mean to say that one type of reflection gives a phase shift of π (radians) while another type of interface gives reflections with phase shift 0 or 2π? Which phase shift corresponds to a fixed-boundary reflection? …a free-boundary reflection?

4. Prove the superposition principle for waves obeying the Wave Equation. What you're actually doing is showing that the Wave Equation is a linear equation. In fact many waves are governed by slightly different equations, but a huge number of these waves still obey the superposition principle because their governing equation is linear! (Hint, create an arbitrary function, by adding together two arbitrary functions that satisfy the Wave Equation, and show that they satisfy the equation)

5. Use Huyghen's principle to derive Snell's Law. See next page for a step-by-step guide, or be brave and jump right in there without looking!

6. Using the formula derived in problem 5, determine what relationship between velocities for a given (just pick one!) approach angle leads to a situation where no angle for the refracted ray can satisfy the equation. You can also go to **http://home.a-city.de/walter.fendt/phe/huygenspr.htm** and check out an animated demonstration of Huyghen's principle applied to this situation. By varying the relative velocities you can see different refracted wavefronts. However, if you push it too far you'll find that you can no longer use Huyghen's method. Describe what I mean by "you can no longer use Huyghen's method". What physical phenomenon does this apparent breakdown of Huyghen's method actually correspond to?

7. Even though swells in open water may be travelling straight north, you may notice that the waves on our westward-facing (mostly) beaches always end up crashing on shore with their wavefronts nearly parallel to shore. In other words, the waves always bend toward shore. Keeping the general pattern of wave behavior described by Snell's law in mind, what does this tell you about the speed of waves in shallow water vs. the speed of waves in deep water?

(5. Step-by-Step guide)

a. Draw a ray representation of some wave propagating from in one medium to the interface of another with some arbitrary angle (~30degrees relative to the interface normal is safe)
b. Draw one planar wavefront, corresponding appropriately to the ray direction and touching the interface already at one point.
c. Copy this picture into a frame next to the original.
d. Use the washers to propagate the wavefront in the new picture forward a distance corresponding to one unit of time. In other words, the larger washer lets you make the wavelets necessary to show how far the wave moves, during one unit of time, in the medium with a faster wave propagation velocity, while the smaller washer is for where the wave is slower. You choose which medium has higher velocity ($v_1 <$ or $> v_2$). (Note that since the first wavefront only touches the second medium at one point, you only need to draw one wavelet, while you will need to draw at least two to propagate the front in the medium where a planar wavefront has already formed.)
e. If you drew the line for the second wavefront in the first material so that it crosses the interface, erase the part that has entered the second material! Why?
f. This new picture represents the new wavefronts that exist at the new time. Is this single propagation of the initial wavefronts enough information to define the ray direction in the second material?
g. Even if you answered yes to "e", copy this picture and then propagate all wavefront one more time in the new frame to show me that you really can use Huyghens principle properly and didn't just get lucky last time.
h. In this third frame, define the rays associated with the wavefronts in each material.
i. Now the hard part, copy only the rays and interface to a new frame and use this new drawing to derive geometrically Snell's law in terms of velocities and the angles, □, the rays make with the interface (surface) normals.

Hints: You will need to define right triangles so that you can use basic trigonometry. You need to set up an equality so the triangles you choose must share some feature so that you know the value represented there is equal for each triangle. Don't forget that the angles of every triangle add to 180º. Keep track of all things you can define as perpendicular.