Foreword

I AM a high school teacher—a teacher of mathematics and science. What I say in this book comes from my classroom and from the classrooms of my colleagues. My data are collected from the daily work of our students—nine years of classwork and homework. Through these data I trace how certain differences between black English vernacular (BEV) and standard English can affect a BEV speaker's concept of certain quantitative relations.

Nothing of what I say is the outgrowth of any theoretical position on black English vernacular. In fact, I didn't even know there was something called Black English when I began to realize that many of the difficulties my students were having were rooted in language. It was the incongruence of the obvious intelligence and determination of these students with the unusual kinds of misunderstanding that persisted in their work that drove me to find answers. What I arrived at is an acute awareness of the function in English of prepositions, conjunctions, and relative pronouns in the identification of quantitative ideas. In this book I show how the misunderstandings that had puzzled me relate to the students' nonstandard uses of certain prepositions and conjunctions that in standard English distinguish certain quantitative ideas, and I show why there is reason to believe that these nonstandard uses are rooted in the grammar of BEV. I emphasize, however, that it is the many similarities between BEV and standard English that make the differences a problem—more of a problem than they would be if the vocabularies and grammars of the languages were totally distinct.

For students whose first language is BEV, then, language can be a barrier to success in mathematics and science. But it doesn't have to be. If we teachers know where the difficulties can arise—which concepts can be misunderstood and in what ways—and if we know what features of BEV can play a part in these misunderstandings, the potential problems can be averted. Avoiding the problems, however, also depends upon our realizing that BEV, like any other language, is rule-governed—it is not
just "bad" English. As Howard Mims, an associate professor of speech
and hearing at Cleveland State University, put it: "A teacher has to
understand [that] it isn't just a matter of a child's leaving s's off words
when he conjugates a verb. It's programmed in his head like a computer:
third person singular doesn't have an s." 2

Unfortunately, even though linguists have for twenty years been docu-
menting the phonological, lexical, and syntactic features that distin-
guish BEV from standard English and have written extensively about
the effect these differences can have on a black child's learning to read,
there are still many—black and white—who resist the possibility that BEV
is anything but badly learned English. 3 This resistance is often com-
ounded with the assumption that anyone who talks about BEV is going
to maintain that speakers of the language should be taught in the lan-
guage. On the other side, there are those who, recognizing BEV as a
language, view the use of it as a civil right—so much so that any study of
the relationship between it and learning is seen as questioning the integ-
ritv of the language. As a result of these positions, investigation into the
ways that the language may be interfering with the academic perfor-
mance of black students is often shunted aside.

I firmly believe that as long as such resistance continues many young
people are going to miss out on much that could otherwise be available
to them. Just recently an associate superintendent in the public schools
of the District of Columbia was quoted as saying, "My position is just
what it was 15 years ago. I'm not going to deal with Black English any
more than I'm going to deal with Governor Wallace's English or with
President Kennedy's English. . . . I'm not going to waste my time with
that." And the principal of a D.C. elementary school recently expressed
a view still typical of many educators when he said. 'They don't speak
'Black English.' They use 'bad grammar.'" 4 4

In 1979, when a federal judge ruled that elementary school teachers
in Ann Arbor could do a better job teaching BEV speakers to master
standard English if the teachers knew more about the features of the
language the children brought to school, emotions boiled over in the
press. Whereas the plaintiffs' case and the judge's decision focused on
strengthening instruction in standard English, many assumed that
requirements in the mastery of standard English were to be weakened.
that by court order black children were to be taught in BEV. 5

In the sixties some of those who did recognize black English vernacular
as a language claimed it to be deficient in the means necessary for
learning concepts and for carrying out logical thought; black children

were thus seen as verbally deprived. The response was vehement, espe-
cially on the part of linguists and anthropologists, with the Linguistic
Society of America endorsing a resolution "stating that no natural lan-
guage has been shown to be superior to another for the expression of
logical thought" (Labov 1982, 186). 6 But the sensitivity remains.

Roger Brown, of Harvard University, identified this problem in a let-
ter (26 August 1981) to Denis Prager, then associate director of the pres-
ident's Office of Science and Technology Policy (OSTP). Commenting
on a research report on BEV and science education that Sara Nerlove,
then of Carnegie Mellon University, and I had prepared for the OSTP
(Nerlove and Orr 1981), Brown wrote:

For some scholars it has become almost axiomatic that one language cannot be
said to be in any way "better" than any other and, in particular, that Black English
must not be thought in any way inferior to Standard English. In many dimen-
sions this is probably true; perhaps all. The motives of those holding this view
are generally admirable. Still, it is sometimes championed as a kind of dogma
forbidding any empirical inquiry and that is wrong.

When I first sought help from linguists, it was immediately assumed
that my quest stemmed from a view of BEV as an inferior language. At
that point I didn't even know enough to realize what was blocking our
exchange. Not until two years later, when I was able to spell out some of
my data, did I begin to get the help I needed.

In his letter, Roger Brown identified a further controversy that my
work has led me into—the controversy over the Sapir-Whorf hypothesis
that the language one speaks may shape the way one thinks:

The report should be taken very seriously. . . Some linguists and psycholin-
guists reading this report might dismiss it out-of-hand because it involves a
version of what is called the Sapir-Whorf Hypothesis which some believe has
been shown to be incorrect. In fact, it has not been shown to be incorrect and, indeed,
has scarcely been studied in any adequate way. 7

Although it is clearly not my purpose to involve myself in the debate
over the relationship between language and thought, I am very aware
that what I document suggests that language may indeed play a part in
shaping conceptual thinking. My data suggest that language may shape
the way one perceives quantitative relations—specifically, that the way a
BEV speaker may understand certain standard English expressions of
quantitative relations can affect his or her understanding of those rela-
tions. That language may affect the way one thinks in mathematics and
science is significant. As Alfred Bloom of Swarthmore College put it:
I see the process of learning to manipulate words and grammatical structures as instrumental to the development of many of the schemas in which we think, especially in highly abstract realms of cognition where non-linguistic experience cannot substitute for linguistic experience in providing direction to cognitive development. (Letter to the author, 17 December 1981; italics added)

I can only hope that the problematic, sometimes volatile, issues that surround what I have to say in this book will not get in the way of what might otherwise be accomplished.

Tragically, as the debate over such issues goes on, disproportionate numbers of young blacks continue to be labeled "handicapped," "learning disabled," or "behavior problems." And many educators, instead of paying attention to documented language differences that may be interfering with the performance of these students in school, continue to think in terms of cultural deprivation and compensatory education. In 1984 the Washington Post reported that although black students accounted for only 14 percent of the total school enrollment in Montgomery County, Maryland, they accounted for 20.2 percent of those in programs for the "mentally retarded," 23.5 percent of those in programs for the "emotionally impaired," and 27 percent of those with "specific learning disabilities." In the same year, a report prepared by the Department of Instructional Services of Fairfax County, Virginia, stated that while black students accounted for only 7.7 percent of the total school population, they accounted for 12.4 percent of those in programs for the "emotionally disturbed," 29.2 percent of those in programs for the "mildly retarded," 11.3 percent of those in programs for the "moderately retarded," and 17.4 percent of those in programs for the "learning disabled." As everyone well knows, such disproportionate distribution is justified by some with the claim that blacks are genetically less intelligent than whites, and is explained by others as reflecting a raft of supposed deficiencies in the home environment of black children. The focus is still on deficiencies, rarely simply on differences that may be interfering with performance.

As Walt Wolfram, one of the early researchers into BEV, observes: "Popular notions about language are so thoroughly entrenched that they're not going to be overcome overnight. We're still confronting the same thinking we encountered 20 years ago. I guess that doesn't say much for the rate of social change." And Orlando Taylor, acting dean of Howard University's School of Communications, identifies the challenge that must be faced:

All you have to do is look at the national statistics on school achievement in language arts for minority children to see the traditional approaches don't work. . . Children who come to school speaking nonstandard English score at or near the bottom. When that happens, you either have to assume there's something innate in blacks that prevents their learning standard English, or something inadequate in teachers, or—the one I argue for—that teachers have in their hands an approach that is inappropriate.

In chapter 1, I outline the circumstances that led me to what I describe in this book, and I introduce the reader to some of the kinds of misunderstanding that my colleagues and I encountered. In chapters 2 through 5, I trace the function in standard English of certain prepositions in the expression of certain quantitative relations and show how the students' misunderstandings of these relations are connected to their nonstandard uses of these prepositions. In chapter 6, I show reason to believe that the students' nonstandard uses of prepositions are related to the grammar of black English vernacular. In chapter 7, I introduce the reader to a kind of nonstandard construction in which the students combine in single statements parts of different ways of expressing ideas in standard English. In chapter 8, I show how the students combine the standard English as and than modes of expressing comparisons and how the resulting combinations are related to a lack of distinction between addition and multiplication and between subtraction and division and thus to a confusion between twice and half. In chapters 9 and 10, I show how the students' nonstandard ways of expressing partitive comparisons are related to their nonstandard perceptions of division, and I explore some speculations about the roots of the students' nonstandard as and than expressions. In the Afterword, I consider the problem of what can be done.

In selecting examples, my primary concern has been to choose those which I heard the writers explain in class or which are very much like those I heard discussed. Thus the source of my explanations is the students themselves. My second concern has been to choose as often as possible those examples that demonstrate misunderstandings that are somewhat isolated as opposed to those that are clearly the products of several misunderstandings embedded in one another. It has therefore not been possible to follow representative students through all the types of misunderstanding I discuss.

I want to thank those who have been essential to the gradual working out of the understanding that I present in this book. The process began...
in our faculty meetings at the Hawthorne School; together we hammered out the germs of what I understand today. Without the dedication, keenness of mind, and perseverance of my colleagues, my thinking could never have developed to the point where it was clear enough for me to take my questions to professional researchers. Sara Nerlove of the National Science Foundation understood my questions and valued them. It was she who moved me from a still somewhat involuted understanding to one that reflected current knowledge and could be communicated to others. I am especially grateful to her for alerting me to the significance of markedness in adjectives. Special thanks go to Rae Alexander-Minter: when my ideas were still clumsy she had a sense of what I was trying to do and brought these ideas to the attention of the president's Office of Science and Technology Policy. Headed at that time by Frank Press, science adviser to President Carter, this office was concerned about the disproportionately small number of minority men and women entering the fields of science and engineering. A subsequent research contract with this office led to the OSTP report on BEV and science education (Nerlove and Orr, 1981); this report has served as the first stage of this book. As linguistic consultant for the research made possible by this contract, Walt Wolfram, of the University of the District of Columbia and the Center for Applied Linguistics, provided me with my first knowledge of BEV grammar and of linguistics in general. I thank him particularly for bringing to my attention the conjunctive use of which, the deletion of subject relative pronouns, and the nonstandard blend of two standard modes of relative-clause formation. To Alfred Bloom of Swarthmore College, I am indebted for his dogged and perceptive pursuit of the possible meanings of twice as small as and twice as less. My thanks go also to Marcia Linebarger, of Swarthmore College and System Development Corporation, for our long discussions about any and some; inevitably her penetrating responses to my many questions sent me back to think again. And finally, I will always be grateful to William A. Stewart, of the City University of New York, for freeing me from the last traces of ethnocentrism and for making available to me unstintingly his keen insights into the linguistic world of a BEV speaker. In particular I am indebted to him for encouraging me to pursue my ideas about the role of negation in the production of the students' nonstandard as and than expressions and especially for the clarity and thoughtfulness with which he considered the details of my understanding as it emerged.
"I Tried to Make Them All Equal"

Assignments were designed to zero in on whatever might lie behind the inordinate failure rate of these students in geometry. One such assignment consisted of thirty-two questions that addressed the concept of quantity along with the notions of adding equal quantities to equal quantities, of subtracting equal quantities from equal quantities, and of doubling or taking halves of equal quantities. Among those who did this assignment during their second week in geometry was a student whom I shall call Jane. Figure 1.1 shows the responses of one of Jane's classmates to three questions in the assignment, and figure 1.2 shows Jane's responses.

Jane, a seventeen-year-old, had spent enough years in school to be in the eleventh grade, but at Hawthorne she had an ungraded status—a designation used for students whose credits are either insufficient in quantity or not in the required distribution for a given grade level. A student with this status can move ahead in one subject area while remaining behind grade level in another.

When she came to Hawthorne as a ninth grader, Jane was, as a result of placement tests, put in a math skills course—one of a series of courses designed by the Hawthorne faculty to address high school students' lack
12. The distance from Washington, D.C., to New York City is equal to the distance from Washington to Cleveland, Ohio. Johnstown, Ohio, is fifty miles further from Washington than Cleveland is. Springsville, New York, is fifty miles further from Washington than New York City is.

a) In the space below, draw a labeled diagram that depicts the distance from Washington to New York City and the distance from Washington to Cleveland.

```
\[\text{DC} \rightarrow \text{NY} \rightarrow \text{SP} \rightarrow \text{NY} \rightarrow \text{CH}\]
```

b) To the diagram you drew for 12a, add whatever is necessary in order to also show the locations of Springsville, New York, and Johnstown, Ohio. Label these locations.

13. The distance from Washington, D.C., to New York City is equal to the distance from Washington to Cleveland, Ohio. Aurora, Ohio, is forty miles closer to Washington than Cleveland is. Lakeland, New York, is forty miles closer to Washington than New York City is.

a) In the space provided below, draw a diagram that depicts the distance from Washington to New York City and the distance from Washington to Cleveland.

```
\[\text{DC} \rightarrow \text{NY} \rightarrow \text{AC} \rightarrow \text{OH} \rightarrow \text{CH}\]
```

b) In the diagram you drew for 13a, show the locations of Lakeland, New York, and Aurora, Ohio.

14. The distance from Washington, D.C., to Chicago, Illinois, is equal to the distance from Washington to Boston, Massachusetts. The distance from Washington to Cleveland, Ohio, is equal to the distance from Cleveland to Chicago. Cleveland is located on the route one travels going from Washington to Chicago. The distance from Washington to New York City is equal to the distance from New York City to Boston. New York City is located on the route one travels going from Washington to Boston.

a) In the space provided below, draw a labeled diagram that depicts the distance from Washington to Chicago and the distance from Washington to Boston.

```
\[\text{DC} \rightarrow \text{CH, IL} \rightarrow \text{CH}\]
\[\text{DC} \rightarrow \text{NY} \rightarrow \text{BOST, MASS.}\]
```

b) In your diagram locate and label the locations of Cleveland, Ohio, and New York City.

FIGURE 1.1 The diagrams of a classmate of Jane's for the Johnstown-Springsville, Aurora-Lakeland, and Chicago-Boston problems
12. The distance from Washington, D.C., to New York City is equal to the distance from Washington to Cleveland, Ohio. Johnstown, Ohio, is fifty miles further from Washington than Cleveland is. Springsville, New York, is fifty miles further from Washington than New York City is.

a) In the space below, draw a labeled diagram that depicts the distance from Washington to New York City and the distance from Washington to Cleveland.

b) To the diagram you drew for 12a, add whatever is necessary in order to also show the locations of Springsville, New York, and Johnstown, Ohio. Label these locations.

13. The distance from Washington, D.C., to New York City is equal to the distance from Washington to Cleveland, Ohio. Aurora, Ohio, is forty miles closer to Washington than Cleveland is. Lakeland, New York, is forty miles closer to Washington than New York City is.

a) In the space provided below, draw a diagram that depicts the distance from Washington to New York City and the distance from Washington to Cleveland.

b) In the diagram you drew for 13a, show the locations of Lakeland, New York, and Aurora, Ohio.

14. The distance from Washington, D.C., to Chicago, Illinois, is equal to the distance from Washington to Boston, Massachusetts. The distance from Washington to Cleveland, Ohio, is equal to the distance from Cleveland to Chicago. Cleveland is located on the route one travels going from Washington to Chicago. The distance from Washington to New York City is equal to the distance from New York City to Boston. New York City is located on the route one travels going from Washington to Boston.

a) In the space provided below, draw a labeled diagram that depicts the distance from Washington to Chicago and the distance from Washington to Boston.

b) In your diagram locate and label the locations of Cleveland, Ohio, and New York City.

FIGURE 1.2 Jane's diagrams for the Johnstown-Springsville, Aurora-Lakeland, and Chicago-Boston problems
of understanding of fractions, decimals, and percents. Her work was so peculiarly poor in this course that the teacher feared the possibility of some kind of handicap. In her second year, however, Jane took two more math skills courses, at the end of which her work in mathematics was viewed as that of a serious and able student. It was in her third year at Hawthorne that she took the geometry course in which the assignment in this example was given. In that same year she received the grade "High Pass" in both English and Latin, and her teachers saw her as a highly intelligent student.

The diagrams in figure 1.1 are what I had, without even thinking about it, assumed the students would draw. The student expresses himself according to convention: a location is represented by a point, and a distance is represented by a line segment. Location and distance are seen as two entities and are represented by two distinct symbols.

In contrast, Jane has used a single symbol, a line segment, to represent both location and distance.

In an attempt to gain some insight into the combination of perceptions and mental processes by which Jane moved from the given information and directions to the drawing of the diagrams, I questioned her in private. I asked her to point out with her pencil where Washington was in her diagram for problem 12. In response, she traced the line segment labeled "Wash." When I asked her to show where Cleveland was, she traced the line segment labeled "Cleveland." And when I asked her to point out New York City, she traced the line segment "NY." I asked her how she decided upon the length for New York City. Repeating the words of the original problem, she responded, "Since the distance from Washington, D.C., to New York City is equal to the distance from Washington to Cleveland, Ohio," ... Oh," she interrupted herself, "I didn't do a very good job at that, but I tried to make them equal." And with that she traced the line segments labeled "Wash." and "Cleveland." I again asked her how she decided on the length for New York City. This time she answered without interrupting herself, "Since the distance from Washington, D.C., to New York City is equal to the distance from Washington to Cleveland, Ohio," I tried to make them all equal."

In problem 12a, when asked to depict distances, Jane responds by drawing line segments that she labels with the names of the locations that the requested distances separate. And in problem 12b, when asked to depict locations, she again responds by drawing line segments, which she labels with the names of the requested locations. When asked orally to identify particular locations in her diagram, she traces the line segments,
saying that she drew these "locations" to be equal in length because the
distances that separate these locations are equal. In problem 12a, because
the verbal representation of two distances utilizes the names of three citi­
ies, she has to draw three equal line segments in order to depict only two
equal distances. And in problem 12b, she has to make the line segments
that she uses to represent cities equal in length because the cities are
equal amounts further from Washington than are New York City and
Cleveland.

Jane's work points up a fundamental question: Does Jane think in
terms of two distinct entities—location and distance—even though she
uses the same symbol for both? Or does she think in terms of only one
entity, which she accordingly represents by a single symbol? And if she
does think in terms of only one entity, is it some kind of hybrid of the
conventional notions of location and distance?

Jane gives us in these diagrams a glimpse into the kinds of mental
images she constructs when she is using the single symbol length, rep­
resenting both location and distance, as a tool with which to think. Even
the diagrams Jane drew for problems 13 and 14 begin to be less incom­
prehensible if one attempts to construct in one's own mind images of the
information given in these problems, while adhering to the requirement
that length be used to represent both location and distance. They can be
seen as possible consequents or extensions of the symbol length when it
is used to represent both location and distance. Consider, for instance,
the mental images one might construct in responding to problem 13:
Two cities, both represented by line segments, are equal distances (that
is, equal line segments) closer to a third city (another line segment) than
two other cities (line segments) are. The first two cities must be repre­
sented by equal line segments because they are equal distances closer to
the third city than the other two cities are. And these other two cities
must also be represented by equal line segments because they are equal
distances from the third city. One can see that Jane's diagrams are not
as lacking in reason as they may initially have appeared to be.

Jane's diagrams suggest the possibility that when words, or symbols,
are used as instruments with which to think, the use in one language of
a single symbol in contexts where a second language requires two or
more can lead a speaker of the first language to arrive at a different
mental construct of some given information from that arrived at by a
speaker of the second language. Or, as in Jane's attempt to handle prob­
lem 14, the result may be an inability to arrive at a workable mental
construct at all.
IN ENGLISH one identifies a distance by the names of the two locations that determine it: these names are held in relation to one another and to the rest of the sentence by certain prepositions. The prepositions most commonly used are from, to, and between, as in the distance from Washington to New York and the distance between Washington and New York. In my experience, students who are in the habit of using these prepositions according to the conventions of standard English do not confuse location and distance; and students who confuse location and distance do not use these prepositions according to those conventions. The combinations from ... to ... and between ... and ... can trigger the need to name the two locations that determine a distance; in a sense each combination sets up two spaces that need to be filled. Without such a “space holder” the structure collapses: the spaces are not there, and what might otherwise be separated is not. For example, a student writes, Aurora Ohio is equal to the distance from Lakeland to Wash. Here two cities that determine a distance have collapsed into one, in the sense that one of the cities has itself become the distance: Aurora has become the distance from Washington to Aurora. Whole phrases and even clauses can collapse for students who are required to speak or write about a number of distances when they are not yet in the habit of using the standard English distance prepositions. And the collapse in sentence structure is often accompanied by a collapse in idea.

“That Distance Will Equal Aurora”

If you subtract half the distance from Washington to Cleveland that distance will equal Aurora and if you subtract half the distance from Washington to New York that distance equals Lakeland. EX. 2.1

“That Distance Will Equal Aurora”

Laying aside, for the moment, the student’s use of half, one can see this sentence as a collapsed version of the kind of sentence shown in figure 2.1. From the perspective of standard English, three uses of the distance from collapse into one in the student’s sentence, where a single from functions as both a locative from and a subtractive from. That is, as shown in figure 2.1, the student’s first from can be understood both as identifying

<table>
<thead>
<tr>
<th>THE LOCATIVE FUNCTION OF A STUDENT’S FROM</th>
<th>Uncollapsed Sentence</th>
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<tbody>
<tr>
<td>If you subtract half the distance from Washington to Cleveland</td>
<td>If you subtract half the distance from Washington to Cleveland</td>
</tr>
<tr>
<td>That distance will equal Aurora...</td>
<td>That distance will equal [the distance from Washington to Cleveland,] Aurora...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>THE SUBTRACTIVE FUNCTION OF A STUDENT’S FROM</th>
<th>Uncollapsed Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you subtract half the distance</td>
<td>If you subtract half the distance</td>
</tr>
<tr>
<td>From Washington (or from Cleveland?) to</td>
<td>From Washington (or from Cleveland?) to</td>
</tr>
</tbody>
</table>
the distance from Washington to Cleveland and as identifying what the student's *half the distance* is to be subtracted from. Similarly, his second *from* can be understood both as identifying the distance from Washington to New York and as identifying what his *half the distance* is to be subtracted from.

Students don't produce sentences of this kind—which clearly are associated with the kinds of concepts that appear in Jane's diagrams and Mary's proofs (see chapter 1)—once they master the *from...to...* and *between...and...* structures of standard English. But, as before, the problem is that it is easy to assume that the uncollapsed version is what the student "means" by what he or she writes; that there is, therefore, no problem since what the student is thinking is clear in spite of the way he or she says it. Note, however, that in this case the student's sentence does not identify whether the Aurora "distance"—if it is a distance and not a location—is from Washington to Aurora or from Cleveland to Aurora. The collapse, therefore, allows room for, and perhaps even helps bring into being, the particular type of misunderstanding that appears in Mary's proof (example 1.3), where the forty miles are seen as separating Washington from Aurora, not Aurora from Cleveland. Some students even understand the forty miles to be the distance from Washington to Aurora as well as the distance from Cleveland to Aurora, with Aurora presumably situated halfway between the other two cities—hence the use of *half* in example 2.1. The collapsing of the expression of distances can thus, at a minimum, support the kind of notion that appears in Jane's diagrams and in Mary's proofs and can even play a part in engendering such notions.
"At a Slower Speed but at the Same Length of Time"

In addition to distance, time, and speed prepositions, there are, in standard English, certain verbs that are used in speaking of one or another of the three quantities. In this sense, spend and take are time verbs, and cover is a distance verb.

These students use speed prepositions for both time and distance:

The distance the faster plane will travel if it travels at the same amount of hrs. as the slower plane. EX. 5.1

Because John is traveling at a slower speed but at the same length of time as Sam. EX. 5.2

... if he had traveled at a distance greater than Sam's. EX. 5.3

The statement is false because the problem does not tell us that John travels at twice the distance by car as he does by train. EX. 5.4

They use both time prepositions and time verbs when speaking of speed:

... the speed he would have to maintain in order to traveled the going trip in the returning trips speed. EX. 5.5

... the speed the train would travel if it had covered the entire distance except y - 30 mls. within the same speed that it covered its actual distance. EX. 5.6

The speed spent by the boat would have to be greater than the speed spent of the boat if the two distances were equal.1 EX. 5.7

"The Fathers Past Age plus x Years Ago"

Right = the speed the going trip would take if its distance equals to2 the going distance. EX. 5.8

They use the distance verb cover when speaking of time or speed:

The speed covered by the boat would be 30 mph. EX. 5.9

To the right says the speed that would have been spent by boat if it had traveled for the hrs. the train covered but still covering the same distance the boat covered. EX. 5.10

The left expression says the no. of hrs. the plane covered on the return trip. The right expression says the no. of hrs. the plane covered on the going trip. If both hrs. covered on the going and returning trip are added together, they will result to the total hrs. covered, 8 hrs. EX. 5.11

They use both time prepositions and time verbs when speaking of distance:

... the hours that the faster train actually travelled during each mile. EX. 5.12

... they will spend less time in each mile. EX. 5.13

... the mph the return traveler would travel if he covered the same distance the going trip took. EX. 5.14

... it would be impossible for the statement to be true if they had to each take half of 255 miles. EX. 5.15
DEFINITE COMPARISONS THAT ARE MULTIPLICATIVE

If \( y \) equals 12 then John would be traveling twice the speed than his mother. Ex. 8.1

With the fewest changes possible, example 8.1 may be expressed in any of the following standard English modes of expression:

\[
\text{NOUN MODE} \quad [A] \text{ twice the speed that his mother traveled at.} \\
[B] \text{ of his mother.} \\
\text{traveled by his mother.}
\]

\[
\text{AS MODE} \quad [A] \text{ twice as fast} \\
[B] \text{ as his mother.}
\]

\[
\text{THAN MODE} \quad [A] \text{ two times faster} \\
[B] \text{ than his mother.}
\]

Thus example 8.1 can be viewed as a composite of part A of the noun mode (noun A, for short) and part B of the than mode (than B):

\[
\text{If } y \text{ equals 12 then John would be traveling} \\
[A] \text{ twice the speed} \\
[B] \text{ than his mother.}
\]

\[
\text{In noun } A, \text{ for example } R18, \text{ above, can be expressed in} \\
\text{standard English as and than modes as follows:} \\
\text{than mode} \quad [A] \text{ ... because the plane covers more m.p.h.} \\
[B] \text{ than he did going.}
\]

\[
\text{as mode} \quad [A] \text{ ... because the plane covers not as few m.p.h.} \\
[B] \text{ as he did going.}
\]

And example 8.18 can be viewed as a composite using these modes:

\[
[\text{than A}] \text{ ... because the plane covers more m.p.h.} \\
[\text{as B}] \text{ as he did going.}
\]
COMPONENTS OF EACH MODE

NOUN MODE. This mode is not used for an indefinite comparison in standard English or in student composites.

\[
\begin{array}{c|c|c}
A & \text{not + as + scalar adj./adv., opposite} & B \\
\text{of adj./adv. in than mode} & \text{noun} & \text{as ...} \\
\text{than mode} & \text{vector adj./adv.} & \text{noun} & \text{than ...}
\end{array}
\]

ALTERNATIVE EXPRESSIONS

AS MODE. The car covers not as many miles as the plane.

THAN MODE. The car covers fewer miles than the plane.

FIGURE 8.4. Alternative standard English modes of expressing an indefinite comparison, one that does not employ a quantifier.

Another student, using less instead of more, also combines than A and as B:

\[\ldots \text{he used less hours to cover the dis. going as the dis. returning.} \text{ ex. 8.19}\]

THAN MODE \[A\] \ldots he used less hours to cover the dis. going \[B\] than the dis. returning.

AS MODE \[A\] \ldots he used not as many hours to cover the dis. going \[B\] as the dis. returning.

And the student composite:

[than A] \ldots he used less hours to cover the dis. going
[as B] as the dis. returning.
<table>
<thead>
<tr>
<th>Nonstandard expression</th>
<th>Standard English equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>two times less than</td>
<td>half of</td>
</tr>
<tr>
<td>two times smaller than</td>
<td>half as large as</td>
</tr>
<tr>
<td>twice as small as</td>
<td>half as large as</td>
</tr>
<tr>
<td>half as small as</td>
<td>half as large as</td>
</tr>
<tr>
<td>twice smaller the number</td>
<td>half the number</td>
</tr>
<tr>
<td>less by two times</td>
<td>half as much as</td>
</tr>
<tr>
<td>half more than</td>
<td>half as much as</td>
</tr>
<tr>
<td>half less than</td>
<td>half as fast as</td>
</tr>
<tr>
<td>twice as slow as</td>
<td>half as fast as</td>
</tr>
<tr>
<td>half as slow as</td>
<td>half as fast as</td>
</tr>
<tr>
<td>two times slower than</td>
<td>half as fast as</td>
</tr>
<tr>
<td>two times younger than</td>
<td>half as old as</td>
</tr>
<tr>
<td>three times less</td>
<td>one-third of</td>
</tr>
<tr>
<td>four times less than</td>
<td>one-fourth of</td>
</tr>
<tr>
<td>four times smaller than</td>
<td>one-fourth of</td>
</tr>
<tr>
<td>four times as small as</td>
<td>one-fourth of</td>
</tr>
</tbody>
</table>

In addition to using these nonstandard modes of expression intact, the students also combine them. Just as they combine the vector adjective of the than mode with the as...as structure of the as mode when the adjective is unmarked (half as larger as), so do they combine these features when the adjective is marked (twice as smaller as and twice as less). And as they do when the adjective is unmarked, they further combine these nonstandard combinations: half as larger as becomes half as larger than and twice as smaller as becomes twice as smaller than.

**Twice and half**

In discussing students' uses of *between*, I introduced examples of students' work to illustrate a kind of confusion between twice and half that appears when their work involves diagrams. In discussing students' perceptions of division, I showed how students treat a 2 in the numerator as a divisor of the fraction and a 2 in the denominator as a multiplier of the fraction, and I presented examples of a corresponding confusion in their use of the words *twice* and *half*. I suggested that the students' lack of familiarity with the standard English usage of *between*, and their perceptions of division, can play a part in sustaining this confusion, in that checks on it that might otherwise be present are missing. I now show why I suspect that this confusion originates in the students' nonstandard ways of expressing comparisons that involve half and twice.

When they use *half* to identify what is conventionally called *twice* and *twice* to identify what is conventionally called *half*, the terms in which they think are the inverse of what they hear and read in the classroom. Moreover, they are in the process of adopting the language of school. A trouble spot is an expression like *half as much as*. The expression is in the vocabulary of both languages—the students' language and the language of school—but with opposite meanings. Sometimes a student will use the expression in its standard sense in reference to an algebraic expression that has a 2 in the denominator, but will nevertheless reason in terms of its nonstandard sense. Sometimes a student will see such an expression in terms of both meanings, shifting from one to the other in the course of figuring out a problem.